Lecture 19

Learning Objectives

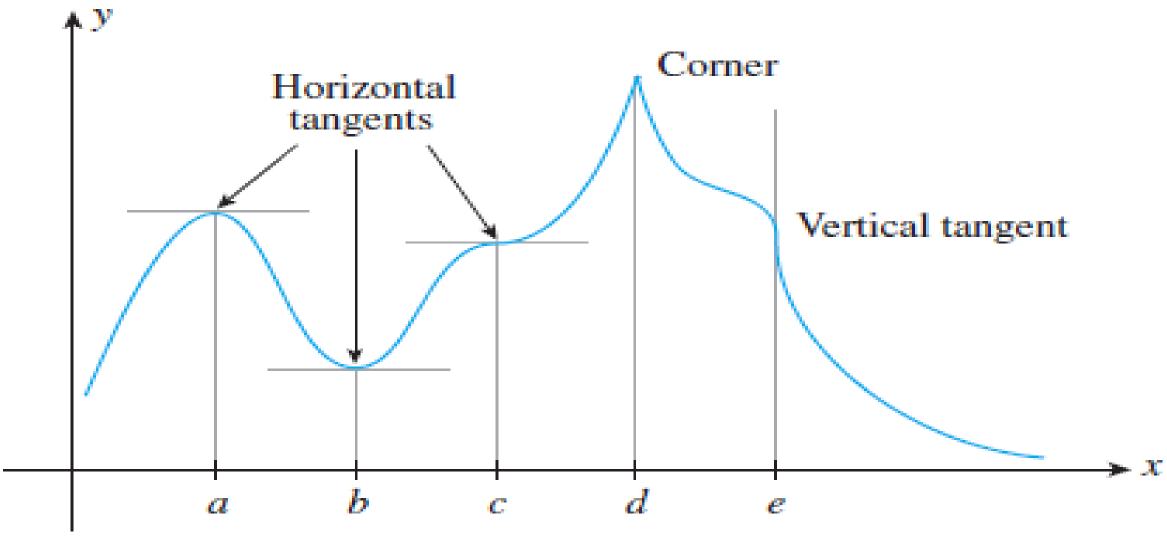
At the end of this class, students should be able to:

- identify absolute maximum and minimum
- solve the problems related to elasticity of demand

19.1 Absolute Maxima and Minima

If $f(c) \geq f(x)$ for all x in the domain of f, then $f(c)$ is called the absolute maximum value of f.
If $f(c) \leq f(x)$ for all x in the domain of f, then $f(c)$ is called the absolute minimum value of f.

In the following figure, if we consider the points in the closed interval $[a, e]$, absolute maximum lies at $x = d$ whereas absolute minimum lies at $x = b$.



We follow the following procedure to find absolute extrema on a closed interval.

Step 1: Check whether the given function f is continuous over the given $[a, b]$.

Step 2: Find the critical values in the interval (a, b).

Step 3: Evaluate f at the endpoints a and b and at the critical values found in step 2.

Step 4: The absolute maximum $f(x)$ on $[a, b]$ is the largest value found in step 3.

Step 5: The absolute minimum $f(x)$ on $[a, b]$ is the smallest value found in step 3.

Illustration 1

Find the absolute maximum and absolute minimum values of $f(x) = x^3 + 3x^2 - 9x - 7$ on the interval [-2, 2].

Solution

Here, $f(x) = x^3 + 3x^2 - 9x - 7$ then

$$f'(x) = 3x^2 + 6x - 9$$

For critical value, setting $f'(x) = 0$

i.e. $3x^2 + 6x - 9 = 0$

or $3(x-1)(x+3) = 0$

$\therefore$ $x = -3, 1$

But x = -3 does not lie in the interval [-2, 2].

Thus, x = 1 is the critical value in the interval [-2, 2].

Evaluating f at the endpoints and critical value, i.e., at $x = -2, 1,$ and 2, we get

$$f(-2) = (-2)^3 + 3\times(-2)^2 - 9\times(-2) - 7 = 15,$$

$$f(1) = (1)^3 + 3\times(1)^2 - 9\times(1) - 7 = -12\ ,$$

$$f(2) = (2)^3 + 3\times(2)^2 - 9\times(2) - 7 = -5$$

Comparing above values, we get the absolute maximum value equal to 15 which exists at $x = -2$ and absolute minimum of -12 which exists at $x = 1$.

19.2 Elasticity of Demand

The elasticity of demand is the negative of the ratio of the relative rate of change of demand to the relative rate of change of price. If price and demand are related by a price demand equation of the form $x = f(p)$ then the elasticity of demand can be expressed as

$$\eta_d = -\frac{p}{x}\frac{dx}{dp}$$

$$= -\frac{p}{f(p)} f'(p) \qquad \left[\because \frac{dx}{dp} = f'(p)\right]$$

η_d is a measure of how much the demand changes for a given change in price.

On the basis of the value of the elasticity, the demand at particular price may be classified into following three categories.

1. For $\eta_d > 1$, demand is said to be elastic.
2. For $\eta_d = 1$, demand is said to be unit elastic.
3. For $\eta_d < 1$, demand is said to be inelastic.

When $\eta_d = 0$, demand is said to be perfectly inelastic and when $\eta_d = \infty$, demand is said to be perfectly elastic.

Illustration 2

Find η_d for the price demand equation $x = f(p) = 1{,}000(40 - p)$. Find and interpret η_d for each of the following price:

a) $p = 8$

b) $p = 20$

c) $p = 30$

Solution

We know that

$$\eta_d = -\frac{p}{x}\frac{dx}{dp}$$

Here, $x = f(p) = 1{,}000(40 - p)$ then $\frac{dx}{dp} = -1{,}000$

Now, $$\eta_d = -\frac{p}{1{,}000(40 - p)}(-1{,}000)$$

or $$\eta_d = \frac{p}{(40 - p)}$$

a) When $p = 8$,

$$\eta_d = \frac{8}{(40 - 8)} = \frac{1}{4} \quad < 1$$

If the \$8 price changes by 1%, then the demand will change by approximately 0.25%.

b) When $p = 20$,

$$\eta_d = \frac{20}{(40 - 20)} = 1$$

If the \$20 price changes by 1%, then the demand will change by approximately 1%.

c) When $p = 30$,

$$\eta_d = \frac{30}{(40 - 30)} = 3 \quad > 1$$

If the \$30 price changes by 1%, then the demand will change by approximately 3%.

Exercise for Reader

1. Find the absolute maximum value and the absolute minimum value, if any, of each function.

   a) $f(x) = x^2 - 2x - 3$ on [-2, 3]

   b) $f(x) = -x^2 + 4x + 6$ on [0, 5]

   c) $f(x) = x^3 + 3x^2 - 1$ on [-3, 1]

   d) $f(x) = \frac{1}{2}x^4 - \frac{2}{3}x^3 - 2x^2 + 3$ on [-2, 3]

2. Acrosonic's total profit (in dollars) from manufacturing and selling *x* units of their model F loudspeaker systems is given by $P(x) = -0.02x^2 + 300x - 200{,}000 \ (0 \le x \le 20{,}000)$. How many units of the loudspeaker system must Acrosonic produce to maximize its profits?

3. Compute the elasticity of demand for the following demand functions.

   a) $x = -1.5p + 25;\ p = 12$

   b) $x = 200 - p^2;\ p = 10$

   c) $x = \frac{2{,}000}{p^2};\ p = 5$

4. Given the price demand equation $p + 0.01x = 50$

   a) Express the demand *x* as a function of the price p.

   b) Find the elasticity of demand, η_d.

   c) What is the elasticity of demand when *p* = 10? If this price is decreased by 5%, what is the approximate change in demand?

   d) What is the elasticity of demand when *p* = 45? If this price is decreased by 5%, what is the approximate change in demand?

   e) What is the elasticity of demand when *p* = 25? If this price is decreased by 5%, what is the approximate change in demand?

5. If $p = 640 - 0.4x$ find the values of *x* for which demand is elastic and for which demand is inelastic.