Lecture 25

Learning Objectives

At the end of this class, students should be able to:

- understand the concept of indefinite integral
- use integration rules
- solve related problems

25.1 Integration

The reverse process of differentiation is known as integration. We use the symbol $\int f(x)\,dx$ to represent the integration of $f(x)$. We write $\int f(x)\,dx = F(x)+c$ if $F'(x) = f(x)$. This integral is called indefinite integral because it involves a constant c that can take on any value. The symbol $\int$ is called an integral sign and the function $f(x)$ is called the integrand. The symbol dx indicates that the integration is performed with respect to the variable *x*. The letter *c* is called the constant of integration.

Note: While finding integral, if $F'(x) = f(x)$, then the integration $\int f(x)\,dx = F(x)+c$ is correct, but if $F'(x)$ is anything other than $f(x)$, we must have committed a mistake.

25.2 Basic Integration Rules

We present some standard rules of integration.

1. The Constant Rule

$\int k\,dx = kx + c$, where *k* is a constant

Illustration 1

Evaluate $\int 20dx$

Solution

$$\int 20dx = 20x + c$$

Where *c* is the constant of integration

2. The Power Rule

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \text{ for all } n \neq -1$$

Illustration 2

Evaluate $\int x^{3/2} dx$

Solution

$$\int x^{3/2} dx \qquad = \frac{x^{3/2+1}}{(\frac{3}{2}+1)} + c$$

$$= \frac{2}{5} x^{5/2} + c$$

3. Constant Multiple of a Function

$\int [k.f(x)]dx = k\int f(x)dx$, where k is a constant

Illustration 3

Evaluate $\int 10x^3 dx$

Solution

$$\int 10x^3 dx = 10\int x^3 dx$$

$$= 10 \times \frac{x^4}{4} + c$$

$$= \frac{5}{2} x^4 + c$$

4. The Logarithmic Rule

$$\int \frac{1}{x} dx = \ln|x| + c \text{ for all } x \neq 0$$

Illustration 4

Evaluate $\int \frac{5}{x} dx$

Solution

$$\int \frac{5}{x}\,dx \quad = 5\int \frac{1}{x}\,dx$$

$$= \frac{2}{5}\ln x + c$$

5. The Exponential Rule

$$\int e^{kx}\,dx = \frac{e^{kx}}{k} + c \text{ for constant } k \neq 0$$

Illustration 5

Evaluate $\int 7e^{2x}dx$

Solution

$$\int 7e^{2x}dx \qquad = 7\int e^{2x}dx$$

$$= 7 \times \frac{e^{2x}}{2} + c$$

$$= \frac{7}{2}e^{2x} + c$$

Exercise for Reader

Evaluate the following integrals.

a) $\int \sqrt{5}dx$

b) $\int \sqrt{x}dx$

c) $\int \frac{10}{\sqrt{x}}dx$

d) $\int \frac{4}{x^3}dx$

e) $\int e^{2x}dx$

f) $\int \frac{1}{5x}dx$

g) $\int e^{-5x}dx$

h) $\int \frac{21}{4x}dx$