Lecture 29

Learning Objectives

At the end of this class, students should be able to:

- evaluate the integrals by using integration by parts

Integration by Parts

Let *u* and *v* be two functions of *x*. then

$$\int u\,v\,dx = u\int v\,dx - \int\left\{\frac{d}{dx}(u)\int v\,dx\right\}dx$$

This is the integral of the product of two functions, and is known as Integration by Parts.

We can express the result as under:

$\int u\,v\,dx$ = (1^{st} function) × (Integral of 2^{nd} function)

– Integral of {(derivative of 1^{st} function) × (Integral of 2^{nd} function)}

where, *u*: 1^{st} function and *v*: 2^{nd} function

How can we select *u* and *v* to make integration by parts work? As a general guideline, we do the following.

First, identify the types of functions occurring in the problem in the order:

Logarithm, Polynomial (or Power of *x*), Radical, Exponential

Second, choose *u* to equal the function whose type occurs first on the list. Then *v* equals the rest of the integrand so that *uv* equals the original integrand. A helpful way to remember the order of the function types that help us choose *u* is the sentence

"Lazy People Rarely Excel."

in which the first letters, LPRE, coordinate with the order and types of functions.

Let us try to understand with the help of the following illustrations.

Illustration 1

Evaluate $\int xe^{x}\,dx$

Solution

The integral contains a polynomial (*x*) and exponential (e^{x}). Thus, let $u = x$ and $v = e^{x}$.

Now

$$\int xe^x\,dx = x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x dx\right\}dx$$

$$= x\times e^x - \int\left\{1\times e^x\right\}dx$$

$$= xe^x - \int e^x\,dx$$

$$= xe^x - e^x + c$$

$$= e^x(x-1)+c$$

Illustration 2

Evaluate $\int x\sqrt{x+1}\,dx$

Solution

The integral contains a polynomial (x) and radical $(\sqrt{x+1})$. Thus, let $u = x$ and $v = \sqrt{x+1}$.

Now

$$\int x\sqrt{x+1}\,dx = x\int\sqrt{x+1}\,dx - \int\left\{\frac{d}{dx}(x)\int\sqrt{x+1}dx\right\}dx$$

$$= x\times\frac{2}{3}(x+1)^{3/2} - \int\left\{1\times\frac{2}{3}(x+1)^{3/2}\right\}dx$$

$$= \frac{2x}{3}(x+1)^{3/2} - \frac{2}{3}\int (x+1)^{3/2}\,dx$$

$$= \frac{2x}{3}(x+1)^{3/2} - \frac{2}{3}\times\frac{2}{5}(x+1)^{5/2}+c$$

$$= \frac{2x}{3}(x+1)^{3/2} - \frac{4}{15}(x+1)^{5/2}+c$$

Illustration 3

Evaluate $\int_0^1 x^2e^x dx$

Solution

First solve it by removing the limits, so

$$\int x^2e^x\,dx = x^2\int e^x\,dx - \int\left\{\frac{d}{dx}(x^2)\int e^x dx\right\}dx$$

$$= x^2\times e^x - \int\left\{2x\times e^x\right\}dx$$

$$= x^2e^x - 2\int xe^x\,dx$$

$$= x^2e^x - 2\left[x\int e^x\,dx - \int\left\{\frac{d}{dx}(x)\int e^x dx\right\}dx\right]$$

$$= x^2e^x - 2\left[x \times e^x - \int\left\{1 \times e^x\right\}dx\right]$$

$$= x^2e^x - 2\left[xe^x - \int e^x\,dx\right]$$

$$= x^2e^x - 2\left[xe^x - e^x\right] + c$$

$$= e^x(x^2 - 2x + 2) + c$$

Now

$$\int_0^1 x^2e^x dx = \{e^x(x^2 - 2x + 2) + c\}\Big|_0^1$$

$$= \left[e^1(1^2 - 2\times 1 + 2) + c\right] - \left[e^0(0^2 - 2\times 0 + 2) + c\right]$$

$$= e - 2$$

Exercise for Reader

Evaluate the following integrals.

a) $\int xe^{x}dx$

b) $\int x^{2}e^{-x}dx$

c) $\int x^{2}\ln x\,dx$

d) $\int_{0}^{1} xe^{x}dx$

e) $\int_{1}^{2} x^{3}\ln x\,dx$

f) $\int_{0}^{5}\ln(x+3)dx$