Lecture 32

Learning Objectives

At the end of this class, students should be able to:

- apply matrix operations
- solve related problems

Matrix Addition

If two matrices have the same order, they may be added together. The result is a new matrix with the same order in which each element is the sum of the corresponding elements of the previous matrices.

Illustration 1

Let A = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$, then find A + B.

Solution

Here A + B = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -1+3 & 4-5 & 2-4 \\ 3+4 & 0+0 & 7+2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & -2 \\ 7 & 0 & 9 \end{bmatrix}$

Matrix Subtraction

Matrix subtraction is like addition. Each element of one matrix is subtracted from the corresponding element of the other. The result is a new matrix with the same order.

Illustration 2

Let A = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$, then find A – B.

Solution

Here A - B = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -1-3 & 4+5 & 2+4 \\ 3-4 & 0-0 & 7-2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 9 & 6 \\ -1 & 0 & 5 \end{bmatrix}$$

Multiplication of a Matrix by a Number

The product of a number k and a matrix A, denoted by kA, is a matrix formed by multiplying each element of A by k.

Illustration 3

Let A = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ then find 5A.

Solution

Here 5A = $5\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$

$$= \begin{bmatrix} 5\times-1 & 5\times4 & 5\times2 \\ 5\times3 & 5\times0 & 5\times7 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 20 & 10 \\ 15 & 0 & 35 \end{bmatrix}$$

Matrix Product

If A is an $m\times p$ matrix and B is a $p\times n$ matrix, then the matrix product of A and B, denoted AB, is an $m\times n$ matrix whose element in the i^{th} row and j^{th} column is the real number obtained from the product of the i^{th} row of A and the j^{th} column of B. If the number of columns in A does not equal the number of rows in B, then the matrix product AB is not defined.

It is important to check sizes before starting the multiplication process.

- If A is an a×b matrix and B is a c×d matrix, then if b = c, the product AB will exist and will be an a×d matrix.
- If b ≠ c, then the product AB does not exist.

The matrix multiplication is NOT commutative, i.e., AB ≠ BA.

Illustration 4

Let X = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ and Y = $\begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 3 & 5 \end{bmatrix}$ then find XY.

Solution

Here XY $= \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 3 & 5 \end{bmatrix}$

$$= \begin{bmatrix} -1\times 2 + 4\times 1 + 2\times 3 & -1\times 1 + 4\times 0 + 2\times 5 \\ 3\times 2 + 0\times 1 + 7\times 3 & 3\times 1 + 0\times 0 + 7\times 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 9 \\ 27 & 38 \end{bmatrix}$$

Exercise for Reader

1. Find the sum, if it is defined:

   a) $\begin{bmatrix} 5 & 7 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 8 & -4 \end{bmatrix}$

   b) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 8 & 9 \\ 4 & 6 & 2 \end{bmatrix}$

2. If A = $\begin{bmatrix} 6 & -1 \\ -3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix}$, find 2A – 3B.

3. Find each product, if it is defined:

   a) $\begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}$

   b) $\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}$

   d) $\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix}$

   e) $\begin{bmatrix} 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$

   f) $\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$

4. Ms. Fong and Mr. Petris are salespeople for a new car agency that sells only two models. August was the last month for this year's models, and next year's models were introduced in September. Gross dollar sales for each month are given in the following matrices:

   August Sales

   $$A = \begin{array}{c} \\ \text{Fong} \\ \text{Petris} \end{array} \begin{array}{c} \begin{array}{cc} \text{Compact} & \text{Luxury} \end{array} \\ \begin{bmatrix} \$36{,}000 & \$72{,}000 \\ \$72{,}000 & \$0 \end{bmatrix} \end{array}$$

   September Sales

   $$B = \begin{array}{c} \\ \text{Fong} \\ \text{Petris} \end{array} \begin{array}{c} \begin{array}{cc} \text{Compact} & \text{Luxury} \end{array} \\ \begin{bmatrix} \$144{,}000 & \$288{,}000 \\ \$180{,}000 & \$216{,}000 \end{bmatrix} \end{array}$$

   For example, Ms. Fong had $36,000 in compact sales in August and Mr. Petris had $216,000 in luxury car sales in September.

a) What were the combined dollar sales in August and September for each salesperson and each model?

b) What was the increase in dollar sales from August to September?

c) If both salespeople receive a 3% commission on gross dollar sales, compute the commission for each salesperson for each model sold in September.