Lecture 30

Learning Objectives

At the end of this class, students should be able to:

- apply various matrix operations
- solve related problems

Matrix Addition

Two matrices A and B can be added if and only if their dimensions (sizes) are the same (i.e. both matrices have the identical number of rows and columns. Addition of matrices is performed by adding elements in corresponding positions. The result is a new matrix with the same order.

Illustration

Let A = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$, then find A + B.

Solution

$$\text{Here } A + B = \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix} + \begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1+3 & 4-5 & 2-4 \\ 3+4 & 0+0 & 7+2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & -2 \\ 7 & 0 & 9 \end{bmatrix}$$

Matrix Subtraction

Two matrices A and B can be subtracted if and only if their dimensions (sizes) are the same (i.e. both matrices have the identical number of rows and columns. Subtraction of matrices is performed by subtracting elements in corresponding positions. The result is a new matrix with the same order.

Illustration

Let A = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$, then find A – B.

Solution

$$\text{Here } A - B = \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix} - \begin{bmatrix} 3 & -5 & -4 \\ 4 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 4+5 & 2+4 \\ 3-4 & 0-0 & 7-2 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 9 & 6 \\ -1 & 0 & 5 \end{bmatrix}$$

Multiplication of a Matrix by a Number
The product of a number k and a matrix A, denoted by kA, is a matrix formed by multiplying each element of A by k.

Illustration

Let A = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ then find 5A.

Solution

$$\text{Here } 5A = 5\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 5\times -1 & 5\times 4 & 5\times 2 \\ 5\times 3 & 5\times 0 & 5\times 7 \end{bmatrix}$$
$$= \begin{bmatrix} -5 & 20 & 10 \\ 15 & 0 & 35 \end{bmatrix}$$

Matrix Product
If A is an $m\times p$ matrix and B is a $p\times n$ matrix, then the matrix product of A and B, denoted AB, is an $m\times n$ matrix whose element in the i^{th} row and j^{th} column is the real number obtained from the product of the i^{th} row of A and the j^{th} column of B. If the number of columns in A does not equal the number of rows in B, then the matrix product AB is not defined.
It is important to check sizes before starting the multiplication process.

- If A is an a×b matrix and B is a c×d matrix, then if b = c, the product AB will exist and will be an a×d matrix.
- If b ≠ c, then the product AB does not exist.

The matrix multiplication is NOT commutative, i.e., AB ≠ BA.

Illustration

Let X = $\begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}$ and Y = $\begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 3 & 5 \end{bmatrix}$ then find XY.

Solution

$$\text{Here } XY = \begin{bmatrix} -1 & 4 & 2 \\ 3 & 0 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 3 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} -1\times 2+4\times 1+2\times 3 & -1\times 1+4\times 0+2\times 5 \\ 3\times 2+0\times 1+7\times 3 & 3\times 1+0\times 0+7\times 5 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 9 \\ 27 & 38 \end{bmatrix}$$

Exercise for Reader

1. Find the sum, if it is defined:

   a) $\begin{bmatrix} 5 & 7 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 8 & -4 \end{bmatrix}$

   b) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 6 & 8 & 9 \\ 4 & 6 & 2 \end{bmatrix}$

2. If A = $\begin{bmatrix} 6 & -1 \\ -3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 5 \\ 7 & 3 \end{bmatrix}$, find 2A – 3B.

3. Find each product, if it is defined:

   a) $\begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}$

   b) $\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}$

   d) $\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix}$

   e) $\begin{bmatrix} 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$

   f) $\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$

4. Suppose the weights (in grams) and lengths (in centimeters) of three groups of laboratory animals are given by matrix *A*, where column 1 gives the lengths and each row corresponds to one group.

$$A = \begin{bmatrix} 12.5 & 250 \\ 11.8 & 215 \\ 9.8 & 190 \end{bmatrix}$$

   If the increase in both weight and length over the next 2 weeks is 20% for group I, 7% for group II, and 0% for group III, then the increases in the measures during the 2 weeks can be found by computing *GA*, where

$$G = \begin{bmatrix} 0.20 & 0 & 0 \\ 0 & 0.07 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

a) What are the increases in respective weights and measures at the end of these 2 weeks?

b) Find the matrix that gives the new weights and measures at the end of this period by computing $(I + G)A$ where I is the 3×3 identity matrix.

5. A dietitian plans a meal around three foods. The number of units of vitamin A, vitamin C, and calcium in each ounce of these foods is represented by the matrix M, where

$$M = \begin{array}{r} \text{Vitamin A} \\ \text{Vitamin C} \\ \text{Calcium} \end{array} \overset{\begin{array}{ccc} \text{Food I} & \text{Food II} & \text{Food III} \end{array}}{\begin{bmatrix} 400 & 1200 & 800 \\ 110 & 570 & 340 \\ 90 & 30 & 60 \end{bmatrix}}$$

The matrices A and B represent the amount of each food (in ounces) consumed by a girl at two different meals, where

$$A = \overset{\begin{array}{ccc} \text{Food I} & \text{Food II} & \text{Food III} \end{array}}{\begin{bmatrix} 7 & 1 & 6 \end{bmatrix}}$$

$$B = \overset{\begin{array}{ccc} \text{Food I} & \text{Food II} & \text{Food III} \end{array}}{\begin{bmatrix} 9 & 3 & 2 \end{bmatrix}}$$

Calculate the following matrices, and explain the meaning of the entries in each matrix.

a) MA^T b) MB^T c) $M(A + B)^T$