Lecture 29

Learning Objectives

At the end of this class, students should be able to:

- understand the concept of matrix
- familiar with various types of matrices

Matrix

Assume that a pharmaceutical company uses three raw materials denoted by R1, R2 and R3 to produce four products S1, S2, S3 and S4. The following table gives the number of units of each raw material which are required for the production of one unit of each products.

Raw Material	S1	S2	S3	S4
R1	2	1	4	0
R2	3	2	1	3
R3	1	4	0	5

This information can be re-written as

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 2 & 1 & 4 & 4 \\ 3 & 2 & 1 & 3 \\ 1 & 4 & 0 & 5 \end{bmatrix} \end{array}$$

This is the matrix representation of the given information. There are three rows and four columns in this matrix.

The matrices are generally denoted by capital letters of alphabets, viz., A, B, C, etc. and the elements are denoted by corresponding small letters of alphabets.

Thus, a matrix A is a rectangular array of elements (numbers or other mathematical objects, e.g. functions) a_{ij}.

Any element (or entry) a_{ij} has two indices (as shown in the following matrix A), a row index i and a column index j.

The number of rows followed by the number of column is called size or order of the matrix. The matrix A is said to have the order or size $m \times n$ (read: m by n) if there are m rows and n columns.

If $m = n$, matrix A is called a *square matrix*.

Two matrices A and B of the same order $m \times n$, are *equal* if corresponding elements are equal, i.e. $a_{ij} = b_{ij}$ for $1 \le i \le m$ and $1 \le j \le n$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Special Matrices

The following are some special types of matrices:

Row and Column Matrices

A matrix having only one row is called a row matrix. For example, the matrices $[1 \quad 7]$ and $[2 \quad 5 \quad 6]$ are row matrices of size 1×2 and 1×3 respectively.

A matrix having only one column is called a column matrix. For example, the matrix $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is a column matrix of size 2×1.

Transpose Matrix

The transpose of one matrix is another matrix that is obtained by using rows from the first matrix as columns in the second matrix. Let A be any matrix then its transpose is denoted by A' or A^T.

For example, let $A = \begin{bmatrix} p & q & r \\ a & b & c \end{bmatrix}$ be a 2×3 matrix.

Then its transpose matrix is given by

$$A^T = \begin{bmatrix} p & a \\ q & b \\ r & c \end{bmatrix}$$ which is a 3×2 matrix.

Symmetric Matrix

If the transpose of a matrix is equal to itself, that matrix is said to be symmetric. For example, the following matrices are symmetric.

$$\begin{bmatrix} a & h \\ h & b \end{bmatrix} \text{ and } \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

are symmetric.

Diagonal Matrix

A diagonal matrix is a special kind of symmetric matrix. It is a symmetric matrix with zeros in the off-diagonal elements. For example, the matrix

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

is a diagonal matrix of size 3×3.

Scalar Matrix

A scalar matrix is a special kind of diagonal matrix. It is a diagonal matrix with equal-valued elements along the diagonal. For example, the matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

is a scalar matrix of size 3×3.

Identity Matrix

A scalar matrix in which all the elements in the main diagonal are one is called an identity matrix. For example, the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is an identity matrix of size 3×3.

Null Matrix

A matrix is said to be null matrix if all the elements are zero. It can be of any size. For example, the matrix

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

is a zero (or null) matrix of size 3×2.

Exercise for Reader

1. Suppose that a manufacturer produces three products, say, *A*, *B*, and *C*. The units of labor and material required are shown below.

Items	Product		
	A	B	C
Labor	10	15	17
Material	25	28	27

   Put the given information in matrix form.

2. The following table shows the requirement of the following three items (in Kgs) at a particular restaurant on following days.

	Mon	Tue	Wed	Thu
Beef	**13**	**9**	**7**	**15**
Chicken	**8**	**7**	**4**	**6**
Vegetable	**6**	**4**	**0**	**3**

   Put the given information in matrix form.