Lecture 18

Learning Objectives
At the end of this class, students should be able to:
- find derivative of trigonometric functions
- find derivative of inverse circular functions

18.1 Derivative of Trigonometric Functions
We use the following formulas for finding the derivative of trigonometric functions.

a) $\frac{d}{dx}(sinx) = cosx$

b) $\frac{d}{dx}(cosx) = -sinx$

c) $\frac{d}{dx}(tanx) = sec^2x$

d) $\frac{d}{dx}(cosecx) = -cosecx.cotx$

e) $\frac{d}{dx}(secx) = secx.tanx$

f) $\frac{d}{dx}(cotx) = -cosec^2x$

Illustration
Find $\frac{dy}{dx}$ when $y = \sin(2x+5)$.
Solution
We have $y = \sin(2x+5)$, then

$$\frac{dy}{dx} = \frac{d}{dx}\{\sin(2x+5)\}$$
$$= \cos(2x+5)\frac{d}{dx}(2x+5)$$
$$= \cos(2x+5) \times 2$$
$$= 2\cos(2x+5)$$

Illustration
Find $\frac{dy}{dx}$ when $y = \sqrt{\sec x}$.
Solution
We have $y = \sqrt{\sec x}$, then

$$\frac{dy}{dx} = \frac{d}{dx}(\sec x)^{1/2}$$
$$= \frac{1}{2}(\sec x)^{-\frac{1}{2}}\frac{d}{dx}(\sec x)$$
$$= \frac{1}{2\sqrt{\sec x}}.\sec x.\tan x$$
$$= \frac{1}{2}\sqrt{\sec x}\tan x$$

Illustration
Find $\frac{dy}{dx}$ when $y = \sqrt{x}.\cos(2x^2+5)$.
Solution
We have $y = \sqrt{x}.\cos(2x^2+5)$, then

$$\frac{dy}{dx} = \sqrt{x}.\frac{d}{dx}\{\cos(2x^2+5)\} + \cos(2x^2+5).\frac{d}{dx}(x^{1/2})$$

$= \sqrt{x}.\{-\sin(2x^2+5)\}.\frac{d}{dx}(2x^2+5) + \cos(2x^2+5).\left(\frac{1}{2}\right).(x^{-1/2})$
$= \sqrt{x}.\{-\sin(2x^2+5)\}.4x + \cos(2x^2+5).\left(\frac{1}{2}\right).\frac{1}{\sqrt{x}}$
$= \frac{1}{2\sqrt{x}}\{-8x^2\sin(2x^2+5) + \cos(2x^2+5)\}$

Illustration
Find $\frac{dy}{dx}$ when $x + y = \cos(x - y)$.
Solution
We have $x + y = \cos(x - y)$
Now differentiating both sides with respect to x, we get

$\frac{d}{dx}(x+y) = \frac{d}{dx}\{\cos(x-y)\}$
or, $\frac{d}{dx}(x) + \frac{d}{dx}(y) = -\sin(x-y)\frac{d}{dx}(x-y)$
or, $1 + \frac{dy}{dx} = -\sin(x-y)\{1 - \frac{dy}{dx}\}$
or, $\frac{dy}{dx} - \sin(x-y)\frac{dy}{dx} = -\sin(x-y) - 1$
or, $\{1 - \sin(x-y)\}\frac{dy}{dx} = -\{\sin(x-y)+1\}$
or, $\frac{dy}{dx} = \frac{-\{\sin(x-y)+1\}}{\{1-\sin(x-y)\}}$
∴ $\frac{dy}{dx} = \frac{\{\sin(x-y)+1\}}{\{\sin(x-y)-1\}}$

Illustration
Find $\frac{dy}{dx}$ when $y = e^{\sin x}$.
Solution
We have $y = e^{\sin x}$, then

$\frac{dy}{dx} = \frac{d}{dx}\{e^{\sin x}\}$
$= e^{\sin x}.\frac{d}{dx}(\sin x)$
$= e^{\sin x}.\cos x$
$= \cos x\, e^{\sin x}$

18.2 Derivative of Inverse Circular Functions

We use the following formulas for finding the derivative of Inverse Circular functions.

a) $\boldsymbol{\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}\ ,\ (|x| < 1)}$
b) $\boldsymbol{\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}\ ,\ (|x| < 1)}$
c) $\boldsymbol{\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}}$
d) $\boldsymbol{\frac{d}{dx}(\mathrm{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}\ ,\ (x > 1)}$
e) $\boldsymbol{\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}\ ,\ (x > 1)}$
f) $\boldsymbol{\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}}$

Illustration
Find $\frac{dy}{dx}$ when $y = \sin^{-1}\sqrt{x}$.

Solution

We have $y = \sin^{-1}\sqrt{x}$, then

$$\frac{dy}{dx} = \frac{d}{dx}(\sin^{-1}\sqrt{x})$$
$$= \frac{1}{\sqrt{1-x}}\frac{d}{dx}(\sqrt{x})$$
$$= \frac{1}{\sqrt{1-x}}.\frac{1}{2}x^{-1/2}$$
$$= \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

Illustration

Find $\frac{dy}{dx}$ when $y = \tan^{-1}\frac{3x+2}{4}$.

Solution

We have $y = \tan^{-1}\frac{3x+2}{4}$, then

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}\frac{3x+2}{4})$$
$$= \frac{1}{1+\left(\frac{3x+2}{4}\right)^2}\frac{d}{dx}\left(\frac{3x+2}{4}\right)$$
$$= \frac{16}{16+9x^2+12x+4}.\frac{3}{4}$$
$$= \frac{12}{9x^2+12x+20}$$

Exercise for Reader

1. Find $\frac{dy}{dx}$ if:
    a) $y = \cot(2x-1)$
    b) $y = \sqrt{\sin 3x}$
    c) $y = (ax-b).\cos 2x^3$
    d) $y = \cos^{-1}\frac{x}{a}$
    e) $x - y = \tan(xy)$
    f) $x = a\sin\theta , \mathrm{y} = \mathrm{a}\cos\theta$
    g) $y = cos(\ln \sec x)$
2. Differentiate $\sin x$ with respect to $\cos x$.