Lecture 9

Learning Objectives:
At the end of this class, students should be able to:

- plot the graph of constant functions
- plot the graph of linear functions
- plot the graph of quadratic functions
- solve related problems

Polynomial Functions
A polynomial function of degree n has the general form

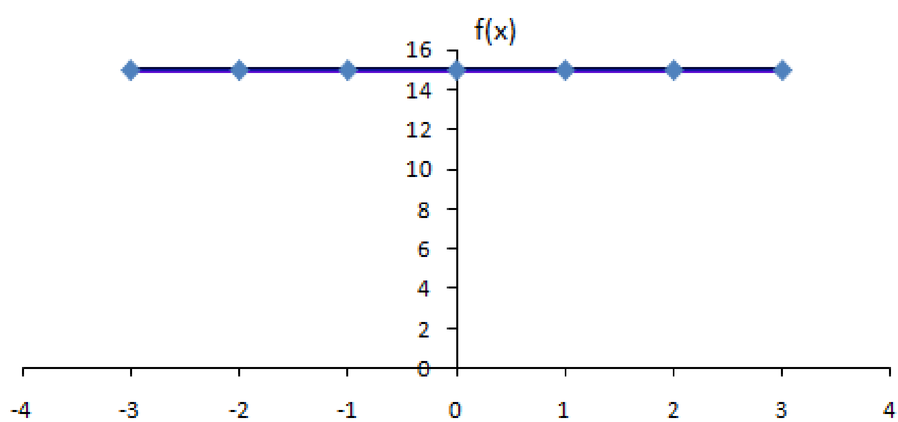
$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

Where a_i are real numbers for each i with $a_n \neq 0$ and n is the positive integer.

The degree of a polynomial is the highest power (exponent) of x in the function.

Constant Functions
A polynomial function of degree zero is called *a constant function*. A constant function has the general form $y = f(x) = a_0$. Where a_0 is a real number. The domain of this function is the set of all real numbers and range is the set consisting of only one number a_o.

For example, $f(x)=15$ is a constant function. The graph of a constant function is a straight-line parallel to x-axis. The following figure represents the graph of the function $f(x)=15$.



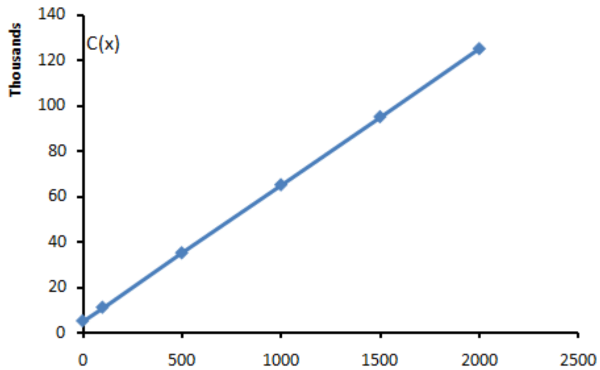
Linear Functions
A polynomial function of degree one is called a linear function. A linear function has the general form $y = f(x) = ax + b$. Where a and b are real numbers and $a \neq 0$. The domain and range of this function are the set of real numbers.

For example, the total cost of owning and operating a drug machinery is given by $C(x) = 0.80x + 20{,}000$.

Illustration
A drug manufacturer's total cost consists of a fixed overhead of $5,000 plus production costs of $60 per unit. Express the total cost as a function of the number of units produced and draw the graph.
Solution
The total cost function is given by

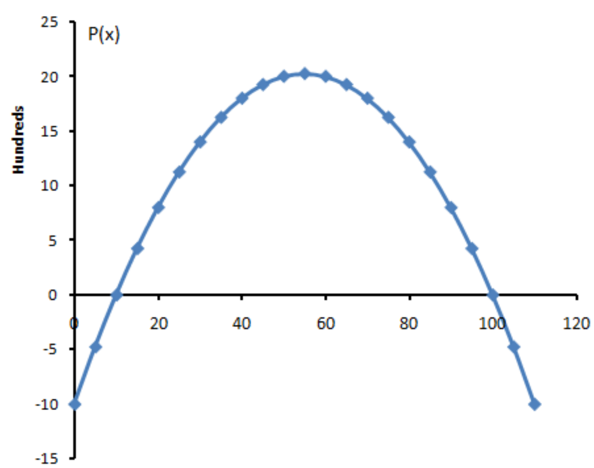
$$C(x) = 60x + 5{,}000$$

Where, x represents the number of units produced. The graph of this function is as follows.



Quadratic Functions
A polynomial function of degree two is called a quadratic function. A quadratic function has the general form $y = f(x) = ax^2 + bx + c$. Where a, b and c are real numbers and $a \neq 0$. The domain of this function is the set of real numbers. For example, the total revenue from the selling a particular drug is given by $R = f(p) = 1{,}500 - 50p^2$, where p is the price stated in dollars.

Illustration
The profit function for a certain medicine is $P(x) = 110x - x^2 - 1000$. Sketch the graph of function. Find the level of production that yields maximum profit, and find the maximum profit.
Solution
The following figure represents the graph of $P(x) = 110x - x^2 - 1000$.



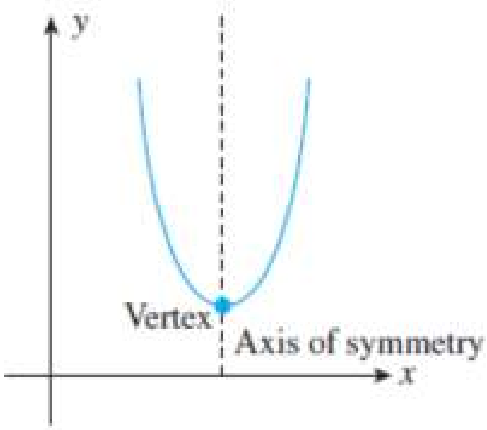
The maximum profit will occur at the value of x that corresponds to the highest point on the profit graph. This is the vertex of the parabola which lies at $x = -b/2a$.

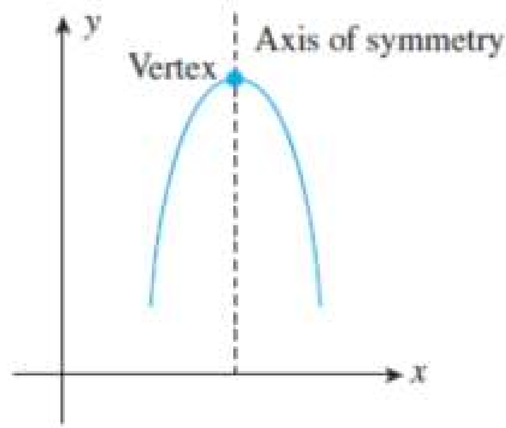
i.e., $x = -110/(2\times-1) = 55$

Thus, profit is maximized when 55 units are produced and sold.
the maximum profit is

$$P(55) = 110\times 55 - (55)^2 - 1000 = 2025$$

In general, the graph of a quadratic function is a parabola that opens upward or downward. Furthermore, the parabola is symmetric with respect to a vertical line called the *axis of symmetry*. This line also passes through the lowest point or the highest point of the parabola. The point of intersection of the parabola with its axis of symmetry is known as the *vertex* of the parabola. The following figures represent these characteristics of the parabola.

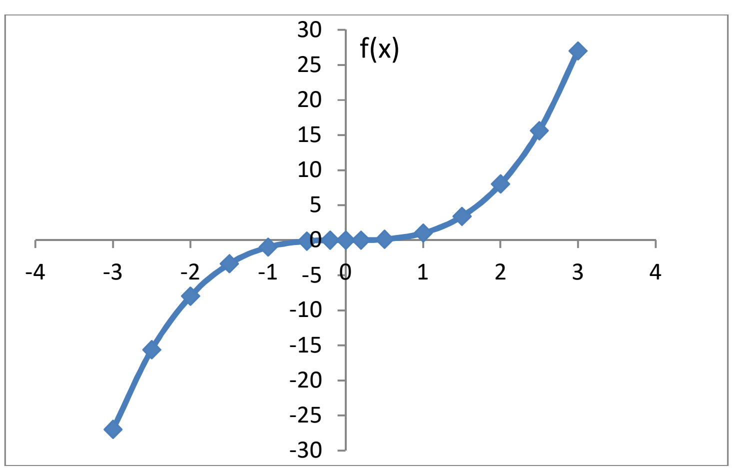




Cubic Function

A polynomial function of degree three is called a cubic function. A cubic function has the general form $y = f(x) = ax^3 + bx^2 + cx + d$. Where a, b, c and d are real numbers, and $a \neq 0$.

For example, $f(x) = x^3$ is a cubic function. The graph of this function is as follows.



Exercise for Reader

1. Puritron, a manufacturer of water filters, has a monthly fixed cost of \$20,000, a production cost of \$20 per unit, and a selling price of \$30 per unit. Find the cost function, the revenue function, and the profit function for Puritron.
2. A manufacturer estimates that it costs \$14 to produce each unit of a particular commodity that sells for \$23 per unit. There is also a fixed cost of \$1,200.
   a) Express the cost $C(x)$ and revenue $R(x)$ as functions of the number of units x that are produced and sold.
   b) What is the profit function for this commodity?
   c) How much profit is generated when 2,000 units of the commodity are produced?
3. The sensitivity S to a drug is related to the dosage x (in milligrams) by $S = 1000x - x^2$. Sketch the graph of this function and determine what dosage gives maximum sensitivity. Use the graph to determine the maximum sensitivity.
4. The demand function for a certain brand of Bluetooth wireless headsets is given by $p = d(x) = -0.025x^2 - 0.5x + 60$ where p is the wholesale unit price in dollars and x is the quantity demanded each month, measured in units of a thousand. Sketch the corresponding demand curve. Above what price will there be no demand? What is the maximum quantity demanded per month?
5. Health care costs in the United States (in billions of dollars) are given by $y = 5.033x^2 + 100.5x + 1378$ where x is the number of years past 2000. Does the model indicate that the cost projections from 2010 to 2015 increase more rapidly than the costs during the years 2005 to 2010?