

Lecture 6

Learning Objectives

At the end of this class, students should be able to:

- identify the number of elements in a particular set

Number of Elements Contained in a Set

Let $A = \{0, 2, 4, 8\}$. There are four elements in the set A . The number of elements contained in a set A is denoted by $n(A)$. Here, $n(A) = 4$.

Let $n(A)$, $n(B)$, $n(C)$, and $n(U)$ denote the number of elements contained in the sets A , B , C , and the universal set (U) respectively, then

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, where $n(A \cap B)$ denotes the number of elements of the set $A \cap B$.
2. $n_o(A) = n(A) - n(A \cap B)$, where $n_o(A)$ denotes the number of elements belonging to A but not B .
3. If A and B are disjoint sets, then $n(A \cup B) = n(A) + n(B)$

$$[\because A \cap B = \phi]$$

4. $n(A \cup B \cup C)$

$$= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

5. $n_o(A) = n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$, where $n_o(A)$ denotes the number of elements belonging to A but not B and C .

Illustration 1

Let $A = \{3, 4, 6\}$ and $B = \{2, 4, 6, 7\}$. Calculate the number of elements in the union $A \cup B$.

Solution

Here, $A = \{3, 4, 6\}$ and $B = \{2, 4, 6, 7\}$ then $A \cap B = \{4, 6\}$.

Thus, $n(A) = 3$ and $n(B) = 4$ and $n(A \cap B) = 2$.

We know that, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\text{i.e. } n(A \cup B) = 3 + 4 - 2 = 5$$

Hence, there are 5 elements in the union.

Illustration 2

A test group of 1000 persons received supplemental doses of vitamin C for a period of 1 year. During that period, it was found that 300 such people had one or more colds, 100 people suffered from influenza, and 80 people suffered from both colds and influenza.

- i) How many persons suffered from either disease?
- ii) How many persons suffered from neither colds nor influenza?
- iii) How many persons suffered from colds but not influenza?

Solution

Here, we have $n(U) = 1000$, $n(C) = 300$, $n(I) = 100$, and $n(C \cap I) = 80$.

i) We know that

$$\begin{aligned} n(C \cup I) &= n(C) + n(I) - n(C \cap I) \\ &= 300 + 100 - 80 \end{aligned}$$

or $n(C \cup I) = 320$

Hence, there are 320 people suffered from either disease.

ii) We know that

$$\begin{aligned} n(\overline{C \cup I}) &= n(U) - n(C \cup I) \\ &= 1000 - 320 = 680 \end{aligned}$$

Hence, there are 680 people suffered from any of diseases.

iii) We know that

$$\begin{aligned} n_o(C) &= n(C) - n(C \cap I) \\ &= 300 - 80 = 220 \end{aligned}$$

Hence, there are 220 people who are suffered from cold only.

Illustration 3

Of the cars sold during the month of July, 90 had air conditioning, 100 had automatic transmissions, and 75 had power steering. Five cars had all three of these extras. Twenty cars had none of these extras. Twenty cars had only air conditioning; 60 cars had only automatic transmissions; and 30 cars had only power steering. Ten cars had both automatic transmission and power steering.

- a) How many cars had both power steering and air conditioning?
- b) How many had both automatic transmission and air conditioning?
- c) How many cars were sold in July?

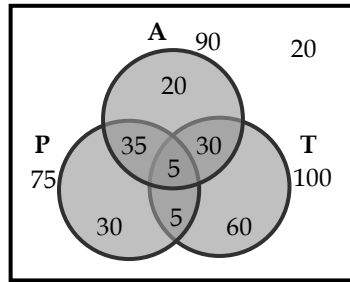
Solution

Let us denote the set of cars having air condition by A, cars having automatic transmission by T, cars having power steering by P, then we have

$$n(A) = 90, n(T) = 100, n(P) = 75, n(A \cap T \cap P) = 5, n(A \cup T \cup P)' = 20,$$

$$n_o(A) = 20, n_o(T) = 60, n_o(P) = 30, n(T \cap P) = 10.$$

The given information is presented in the following Venn diagram.



From the Venn diagram,

$$n(P) = n_o(P) + n(P \cap A) + n(P \cap T) - n(A \cap T \cap P)$$

i.e. $75 = 30 + n(P \cap A) + 10 - 5$

or $n(T \cap A) = 40$

Hence, there were 40 cars which had both power steering and air conditioning.

Again,

$$n(T) = n_o(T) + n(T \cap A) + n(T \cap P) - n(A \cap T \cap P)$$

i.e. $100 = 60 + n(T \cap A) + 10 - 5$

or $n(T \cap A) = 35$

Hence, there were 35 cars which had both automatic transmission and air conditioning.

Now,

$$\begin{aligned} n(U) &= 20 + 35 + 5 + 30 + 30 + 5 + 60 + 20 \\ &= 205 \end{aligned}$$

Thus, 205 cars were sold in July.

Exercise for Reader

1. In a study of ABO blood groups, 6000 individuals were tested. 2527 had the antigen A, 2234 had the antigen B and 1816 had no antigen. How many individuals had both antigens?
2. A survey of 500 high school seniors was conducted to determine the extent to which child abuse exists. It was found that 50 respondents recalled having physically abused by their fathers, 60 recalled having been physically abused by their mothers, and 20 recalled having been physically abused by both parents. How many respondents were:
 - a) Physically abused?
 - b) Not physically abused?
3. Of a group of 200 students, 100 are interested in music, 70 are interested in photography, and 40 like swimming; furthermore 40 are interested in both music and photography, 30

are interested in both music and swimming, 20 are interested in both photography and swimming and 10 are interested in all three activities. Find the number of students that are interested in

- a) Photography but not music and swimming.
 - b) Music only.
 - c) Swimming only.
 - d) Exactly one activity.
 - e) At least one activity.
 - f) None of the activities.
4. A cancer research team working with the support of the department of health has gathered statistics related to the deaths of 20,000 cancer victims. Extensive data have been gathered regarding the health histories and living habits of both the victims and their relatives. Three significant variables appear to be associated with victims of cancer: regular smoking, moderate to heavy drinking, and age of 35 or more. The following data were gathered on 20,000 victims.

14,500 victims regularly smoked.

12,500 victims were moderate to heavy drinkers of alcohol.

15,000 victims were 35 or more years of age.

11,000 were smoked regularly and were moderate to heavy drinkers.

12,000 victims smoked regularly and were 35 or older.

10,000 victims were moderate to heavy drinkers and were 35 or older.

10,000 victims had all three characteristics.

Determine the number of victims:

- i) Who were only regular smokers?
 - ii) Who were only moderate to heavy drinkers and having age 35 or older?
 - iii) Who had non-of these characteristics?
5. A group of 100 people was asked if they had purchased three different brands of antibiotics (ciprofloxacin): Ciprobid, Proxin and Cicin. The following data were gathered.

18 people had purchased Ciprobid.

22 had purchased Proxin.

15 had purchased Cicin.

5 had purchased both Ciprobid and Proxin.

8 had purchased both Ciprobid and Cicin.

6 had purchased Proxin and Cicin.

2 had purchased all three.

- i) How many people had purchased only Cicin?

- ii) How many people had purchased Ciprobid and Proxin only?
- iii) How many people had purchased at least one brand of antibiotic?
- iv) How many people had not purchased any of three?