Lecture 5

Learning Objectives
At the end of this class, students should be able to:

- represent the set
- plot Venn-diagram
- apply various set operations

5.1 Sets

A set is a well-defined collection of objects. Well defined means that there exists a rule with the help of which it is possible to decide if a given object belongs to that collection or not. Thus, every collection is not a set.

The following are the examples of the set:

- The set of pharmaceutical companies in a particular province.
- The set of medicines in a particular drug store.

The objects that belong to a set are called elements of the set. The elements of a set are denoted by small letters whereas the sets are denoted by capital letters.

We use the symbol $\in$ (belongs to) to indicate the set membership, and non-membership is indicated by $\notin$ (does not belong to). If an element x belongs to a set A, we use the notation $x \in A$. Similarly, the notation $y \notin A$ indicates that an object y is not a member of set A.

Two sets A and B are equal if both have the same elements. This is denoted as A = B. If they are unequal, it is expressed as A $\neq$ B**.**

If every element of a set A is also an element of set B, then A is called a subset of B. This is also expressed as: A is contained in B, or B contains A. This is expressed symbolically as A $\subseteq$ B. For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3, 4, 5, 6\}$, we can say $A \subseteq B$.

A subset X is a proper subset of a set Y if X is subset of Y and $X \neq Y$, written as $X \subset Y$. ϕ is a proper subset of any non-empty set. For example, let $X = \{1, 2, 5\}$ and $Y = \{1, 2, 3, 5, 7\}$ then we can say X is subset of Y; moreover, X is a proper subset of Y.

According to the number of elements, sets can be put on two categories. Sets that contain definite number of elements are called finite sets. Sets that contain unlimited number of elements are called infinite sets.

A set which contains only one element is called singleton set. If there is no element in a set, it is called null or empty set, denoted by ϕ or $\{\}$.

A Universal Set is the set of all elements in a particular context, denoted by capital U.

5.2 Set Representation

A set may be represented in different ways. The following are most common methods of describing a set.

Listing Method

According to this method, the elements are listed, separated by commas and enclosed in braces {}. For example, if we denote the set of letters in the word 'mathematics', then it is written as $M = \{m, a, t, h, e, i, c, s\}$.

Note that changing the arrangement of the elements of a set does not change the set. For example, sets $\{a, b, c\}$ and $\{b, a, c\}$ represent the same set. Also, repetition of elements of a set does not change the set.

This method is convenient when the number of elements in a set is small or when it is not easy or possible to articulate a property defining membership in the set.

Descriptive Method

According to this method, the set is defined by stating a property required for membership in the set. We use symbol to denote an arbitrary element of the set. For example, the set of letters of the word 'mathematics' can be written as

$$M = \{x\text{: } x \text{ is a letter of the word 'mathematics'}\}.$$

The 'x' to the left of the '**:**' indicates the general notation for an element of a set, the expression to the right states the condition(s) required of an element for membership in the set.

5.3 Venn Diagram

It is a pictorial representation of the relationship among the sets. Venn diagram utilizes rectangular and circular (or elliptical) areas to represent sets. A large rectangular (or square) area is used to designate the universal set and circular (or elliptical) areas inside the rectangle represent all other sets.

Illustration 1

Sixteen people were surveyed regarding their attitudes about nuclear power. The results of the survey are

Person	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Response	f	a	a	f	f	a	i	f	i	a	f	i	a	f	f	i

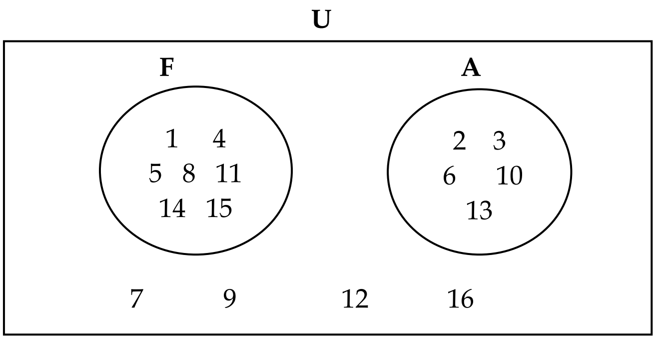
Legend: f = For, a = Against, i = Indifferent

Construct a Venn diagram that summarizes how these persons respond.

Solution

Let F and A represent the sets of people who have expressed their views for and against the nuclear power respectively. Let U represents the universal set. We use circles to represent subsets and rectangle to represent universal set. There is no element common to both the sets F and A, such sets are called disjoint sets.

The relationship between the given sets U, F, and A is shown in the following Venn diagram.



5.4 Set Operations

There are four basic set operations: union, intersection, difference, and complementation. The results of these operations are again sets.

Union of Sets

The union of two sets is a set that consists of all elements found in either set or both sets. The union of sets A and B is denoted by $A\cup B$, and defined by

$$A\cup B = \{x: x \text{ belongs to } A \text{ or } B \text{ or both}\}$$

$$= \{x: x \in A \text{ or } x \in B\}$$

Illustration 2

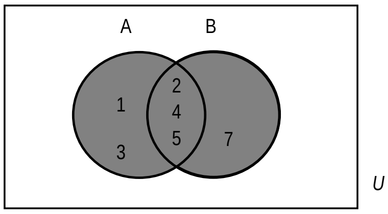
If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 5, 7\}$, then find $A\cup B$ and represent it in a Venn diagram.

Solution

Since $A\cup B$ consists of all the elements that belong to either A or B or both, therefore,

$$A\cup B = \{1, 2, 3, 4, 5, 7\}$$

The shaded region in the following Venn diagram represents $A\cup B$.



Intersection of Sets

The intersection of two sets is the set consisting of all elements that belong to both the sets. The intersection of sets A and B is denoted by $A\cap B$, and defined by

$$A\cap B = \{x: x \in A \text{ and } x \in B\}$$

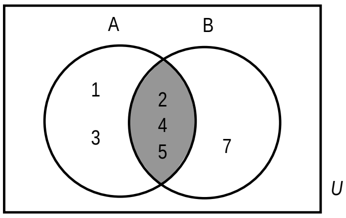
Illustration 3

If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 5, 7\}$, then find $A\cap B$ and represent it in a Venn diagram.

Solution

Since $A \cap B$ consists of all the elements that belong to A and B both, therefore, $A \cap B = \{2, 4, 5\}$

The shaded region in the following Venn diagram represents $A \cap B$.



Difference of Two Sets

The difference of two sets A and B is the set of all elements of A that do not belong to B, denoted by $A - B$, and defined by

$$A - B = \{x: x \in A, x \notin B\}$$

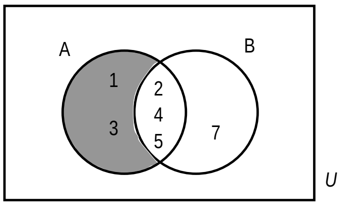
Illustration 4

If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 5, 7\}$, then find A – B and represent it by a Venn diagram.

Solution

Since $A - B$ consists of all the elements that belong to set A but not B, therefore, $A - B = \{1, 3\}$.

The shaded region in the following Venn diagram represents $A - B$.



Complement of a Set

The complement of a set A is the set of all the elements of universal set which do not belong to A, denoted by A^C or A', and defined by

$$A' = \{x: x \in U, x \notin A\}, \text{ or } \quad A' = U - A$$

Illustration 5

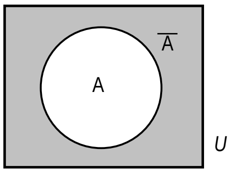
If $U = \{1, 2, 3, \dots, 9\}$ and $A = \{2, 4, 8\}$, then find A' and represent it in a Venn diagram.

Solution

Here $U = \{1, 2, 3, \dots, 9\}$ and $A = \{2, 4, 8\}$

Thus, $A' = \{1, 3, 5, 6, 7, 9\}$.

The shaded region in the following Venn diagram represents A'.



Summary of Set Operations

S. N.	Rule	Union	Intersection
1	Closure	$A \cup B$ is a set	$A \cap B$ is a set
2	Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
3	Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
4	Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
5	Idempotent	$A \cup A = A$	$A \cap A = A$
6	Complement	$A \cup A^c = U$	$A \cap A^c = \phi$
7	De Morgan's Law	$(A \cup B)^c = A^c \cap B^c$	$(A \cap B)^c = A^c \cup B^c$
8	Identity	$A \cup \phi = A$ $A \cup U = U$	$A \cap \phi = \phi$ $A \cap U = A$

Exercise for Reader

Let the sets: U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 4, 5, 6}, B = {0, 2, 5}, and C = {3, 5, 6, 9, 10}. Then find:

a) $B \cup C$ b) $A \cap B$ c) $A \cap (B \cup C)$ d) $A \cup (B \cap C)$ e) $A - B$

f) $\overline{A} \cap A$ g) $\overline{A \cap B}$ h) $\overline{A \cup B \cup C}$ i) $A \cap \overline{B} \cap \overline{C}$