Lecture 3

Learning Objectives

At the end of this class, students should be able to:

- solve (2 × 2) System of equations
- solve (3 × 3) System of equations

3.1 Elimination Procedure

In elimination technique, we apply algebraic operations to eliminate one or more variables from the given system in order to get the solution of the system.

(2 × 2) System

The eliminating procedure can be generalized as follows for a (2 × 2) system equations:

1. Multiply (if necessary) the equations by constants so that the coefficients on one of the variables are the negatives of one another in the two equations.

2. Add the two resulting equations. The following three situations may arise.

(a) If adding the equations results in a new equation having one variable, there is a unique solution to the system. Solve for the value of the remaining variable, and substitute this value back into one of the original equations to determine the value of the variable that was originally eliminated.

(b) If adding the equation results in an identity, i.e., an equation that is always true, such as 0 = 0 or 4 = 4, the two original equations are equivalent to each other and there are an infinite number of solutions to the system.

(c) If adding the equations results in a false statement, say, 0 = 5, the equations are inconsistent and there is no solution set.

We can summarize the procedure in the following tree diagram

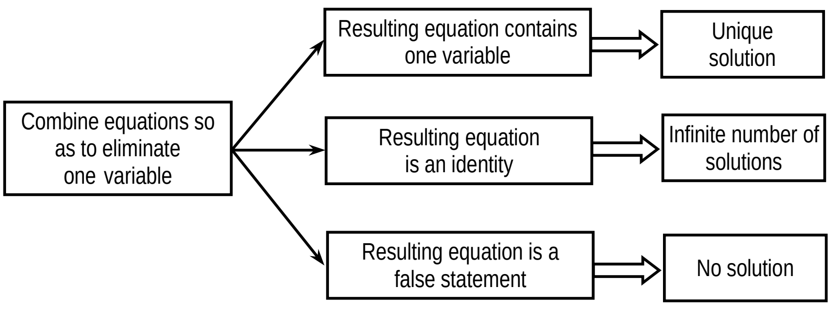


Illustration 1

Solve the following system of equations by elimination method:

$2x + 5y = 11$

$3x + 4y = 13$

Solution

We have

$2x + 5y = 11 \quad \ldots \text{(i)}$

$3x + 4y = 13 \quad \ldots \text{(ii)}$

Multiplying equation (i) by 3 and equation (ii) by – 2 and then adding, we get

$$\begin{array}{rcrcr} 6x & + & 15y & = & 33 \\ -6x & - & 8y & = & -26 \\ \hline & & 7y & = & 7 \end{array}$$

or $y = 1$

Now substituting $y = 1$ in equation (i), we get

$2x + 5 \times 1 = 11$

or $2x = 11 - 5$

or $x = 3$

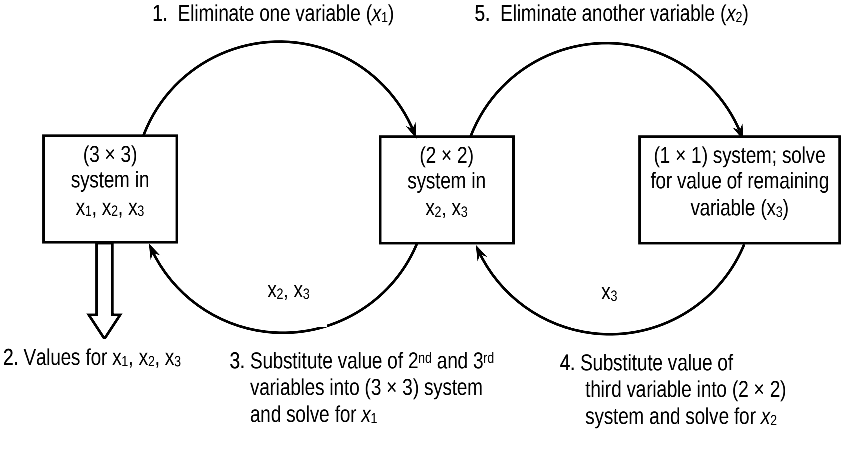
Thus, the required solution of the given system is $x = 3$, $y = 1$.

(3 × 3) System

The elimination procedure for (3 × 3) systems is similar to that of (2 × 2) systems. Here we start with the (3 × 3) system and try to reduce this to an equivalent system having two variables and two equations. After eliminating one of three variables, the same procedure as used for (2 × 2) systems is employed to eliminate a second variable, which results it in a (1 × 1) system.

After solving for the remaining variable, its value has to be substituted sequentially back through the (2 × 2) system and finally the (3 × 3) system to determine the values of the other two remaining variables.

We can summarize the procedure in the following figure.



Exercise for Reader

1. Solve the following system of equations by the elimination procedures.

   a) $3x + 4y = 12$

      $4x + 3y = 12$

   b) $4x + 2y = 10$

      $2x + y = 5$

   c) $x + 2y = 5$

      $x + 2y = 8$

2. Solve the following system of equations.

   a) $x + y + z = 6$

      $2x – y + 3z = 4$

      $4x + 5y – 10z = 13$

   b) $– 2x + y + 3z = 12$

      $x + 2y + 5z = 10$

      $6x – 3y – 9z = 24$

   c) $x + y + z = 20$

      $2x – 3y + z = - 5$

      $6x – 4y + 4z = 30$