Lecture 2

Learning Objectives
At the end of this class, students should be able to:

- plot the graph of a linear equation
- solve (2 × 2) system of equations by graphical method

2.1 Graph of a Linear Equation

We use cartesian plane to plot the graph of a linear equation. A linear equation involving two variables x and y has the standard form ax + by = c, where a, b, and c are real numbers and a and b cannot both equal to zero. For example, 2x + 3y = 6 is a linear equation involving two variables x and y. If we put any value for x in the given equation, we get corresponding value of y or vice-versa. For example, if we put x = 2 in the equation 2x + 3y = 6, we get y = 2/3.

Illustration 1

Plot the graph of the equation: $2x + 4y = 8$.

Solution

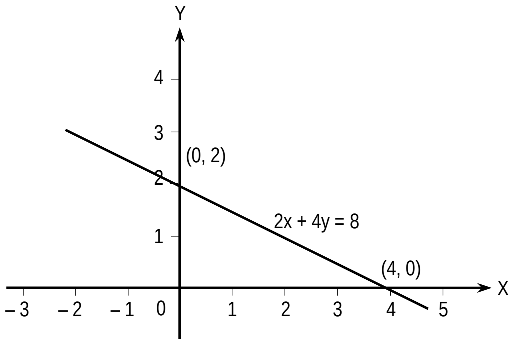
The given equation $2x + 4y = 8$ represents a linear equation in two variables. The graph of this equation is a straight line. The graph of this equation is found by first identifying any two pairs of values for x and y which satisfy the equation.

Note: *Aside from the case where the right side of the equation equals 0, the easiest points to identify are those found by setting one variable equal to 0 and solving for the value of the other variable. That is, let x = 0 and solve for the value of y; again, let y = 0 and solve for the value of x.*

When we put $x = 0$ in the equation $2x + 4y = 8$, the corresponding value for y is 2, and when we put $y = 0$, the corresponding value of x is 4.

Thus (0, 2) and (4, 0) are two points through which the line passes.

A straight line has connected the two points, and the line has been extended in both directions.



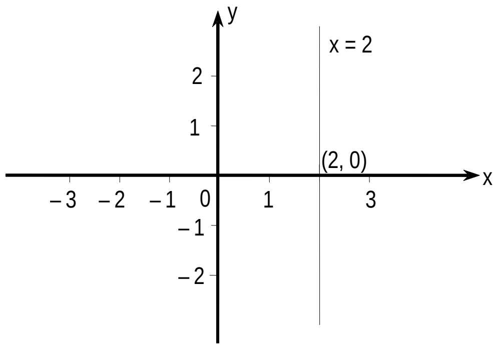
Note: The graph of the equation $x = a$ is the vertical line which crosses the x-axis at $x = a$. Similarly, the graph of the equation $y = b$ is the horizontal line which crosses the y-axis at $y = b$.

Illustration 2

Plot the graph of equation $x = 2$.

Solution

The graph of the equation $x = 2$ is a straight line passing through the point (2, 0) and parallel to y-axis.



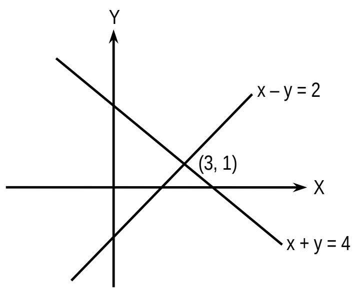
2.2 System of Linear Equations

A system of equations is a set consisting of more than one equation. In solving system of equations, we are interested in identifying values of the variables that satisfy all equations in the system at the same time. For this reason, the group of equations that we solve is often called simultaneous system of equations. Here, we shall study two different techniques: graphical and elimination for finding solution of a system. While solving for the values of the variables which satisfy all the equations, we will check whether there exists any common point in the lines representing the equations.

2.3 Graphical Solution Procedure

Here, we talk about (2 × 2) system of equations, i.e., system of equations represented by two straight lines in two dimensions. For (2 × 2) system equations three different types of solution sets might exist. It is difficult to plot the graph of the equation in three variables, so we shall only discuss about the elimination procedure in that case.

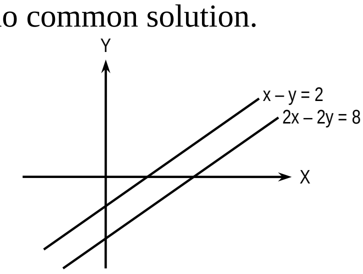
1. When the two lines graphed in a plane, if they intersect, or cross one other, the coordinates of the point of intersection represents the solution for the equations represented by these two lines. When there is just one pair of values for the variables that satisfy the system of equations, the system is said to have a unique solution.



For example, if we solve the equations: $x + y = 4$, and $x - y = 2$, we get unique solution $x = 3$ and $y = 1$. The graph is shown in the figure.

2. When the two lines are parallel to each other, i.e., they never meet, there will be no set of values of x and y both equations. In other words, they have no common solution.

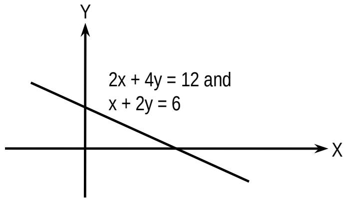
For example, if we solve the equations $x - y = 2$, and $2x - 2y = 8$, we get no solution. The equations like $x - y =$



2, and $2x - 2y = 8$ are called inconsistent system of equations. The graph is shown in figure the figure.

3. The last possibility is the case when the graph of both the equations coincides, i.e., one fits exactly on the other. In this case, the set of values of x and y which satisfies the first equation satisfies the second also. In this case, we will get infinite number of solutions. The graph is as follows:

For example, if we solve the equations: $2x + 4y = 12$, and $x + 2y = 6$, we get infinite number of solution. The graph is shown in the figure.



Another way of summarizing the three conditions is as follows:

In (2 × 2) system of linear equations let m_1 and m_2 represents the respective slopes of the two lines and d_1 and d_2 represents the respective y intercepts.

- There is a unique Solution to the system if $m_1 \neq m_2$.
- There is no Solution to the system if $m_1 = m_2$, but $d_1 \neq d_2$.
- There are infinite number of Solutions to the system if $m_1 = m_2$ and $d_1 = d_2$.

Exercise for Reader

1. Plot the graph of the following equations.

   a) $2x + 3y = 12$

   b) $x - 2y = 0$

   c) $x - 2y = 5$

2. Solve the following system of equations by the graphical procedures.

   a) $3x + 4y = 12$

      $4x + 3y = 12$

   b) $4x + 2y = 10$

      $2x + y = 5$

   c) $x + 2y = 5$

      $x + 2y = 8$