Lecture 1

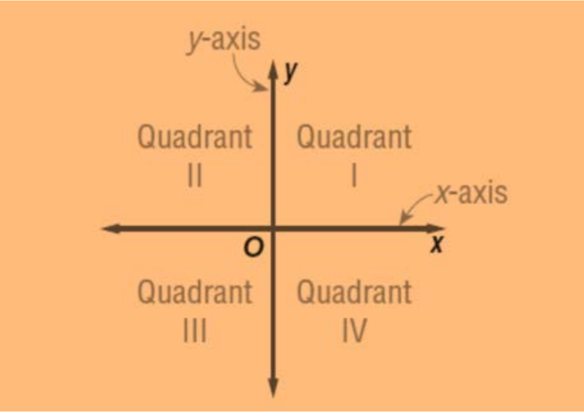
Learning Objectives
At the end of this class, students should be able to:

- locate the points in the cartesian plane
- find an equation of straight line
- solve linear equations in one variable
- solve linear inequalities in one variable

1.1 Cartesian Co-ordinate System

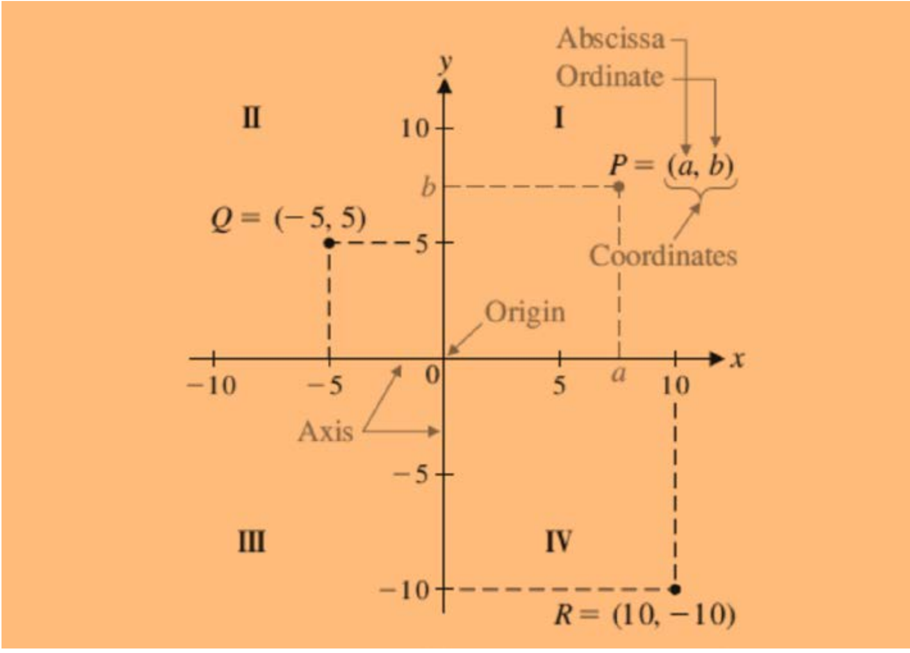
When a horizontal (left to right) number line is crossed with a vertical (up and down) number line, the result is a two-dimensional coordinate plane. Thus, a plane in which a horizontal number line and a vertical number line intersect at their zero points is called a coordinate plane.

The number lines are called axes. The horizontal number line is the *x*-axis, and the vertical number line is the *y*-axis. The plane is divided into four regions, called quadrants. Each quadrant is named by a Roman numeral, as shown in the following diagram.



We use Cartesian coordinates to pinpoint where we are on a coordinate plane. Using Cartesian Coordinates, we mark a point on a plane by how far along and how far up it is. In the following figure, for point P, the vertical line intersects the horizontal axis at a point with coordinate *a*, and the horizontal line intersects the vertical axis at a point with coordinate *b*. These two numbers, written as the ordered pair (*a*, *b*) form the coordinates of the point P. Thus, we can say, the ordered pair (*a*, *b*) represents the point *P* that is located *a* units along the *x*-axis and *b* units along the *y*-axis. The first coordinate, *a*, is called the abscissa of P; the second coordinate, *b*, is called the ordinate of P.

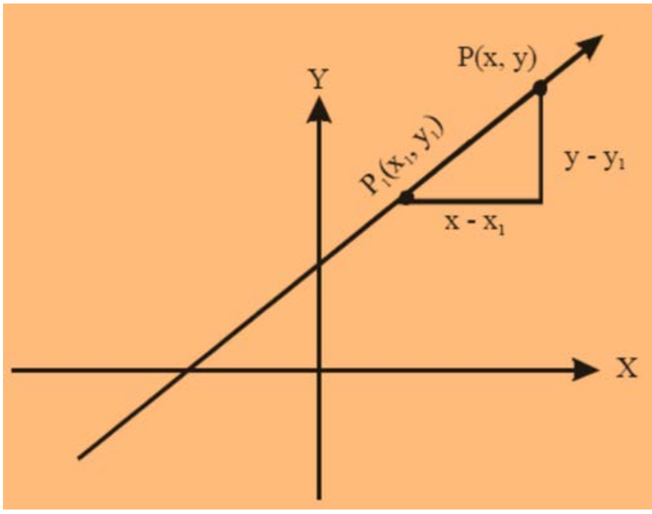
Similarly, the abscissa of Q is -5 and the ordinate of Q is 5. The coordinates of a point can also be referenced in terms of the axis labels. The x coordinate of R is 10, and the y coordinate of R is -10. The point with coordinates (0, 0) is called the origin.



1.2 Straight Line

The line which passes through two points in such a way that the length of the segment between the points is a minimum.

Suppose a line having slope m and passing through a given point (x_1, y_1) as shown in the following figure.



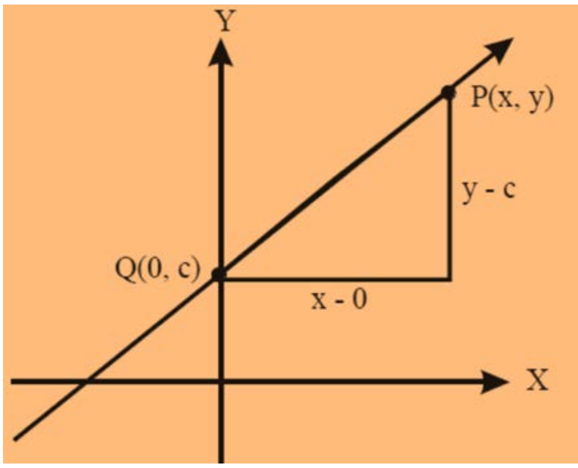
If P(x, y) is any other point on the line, then the slope of the line is

$$\frac{(y - y_1)}{(x - x_1)} = m$$

That is, $y - y_1 = m(x - x_1)$

Which is known as point slope form for a straight line.

Suppose a line having slope m, passing through a given point on the y-axis having coordinates (0, c) as show in following figure.



Substituting (0, c) in the point slope form of a straight line, we get

$$y - c = m(x - 0)$$

or, $$y = mx + c$$

This equation is called slope intercept form of a straight line.

Suppose a line is passing through the points (x_1, y_1) and (x_2, y_2), then slope of the line is

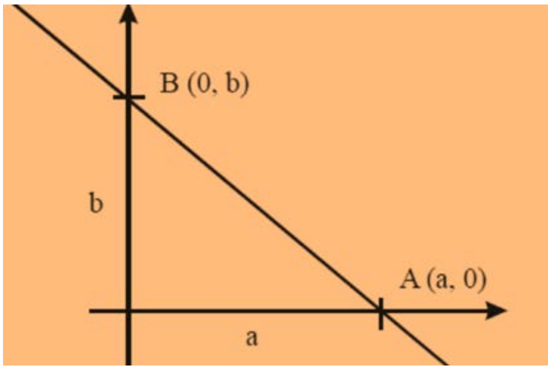
$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Substituting the value of m in the equation $y - y_1 = m(x - x_1)$, we get

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

This is known as the two-point form of the straight line.

Suppose a and b are the x and y-intercepts of a straight line of point A and B respectively. Then the coordinates of point A and B are $(a, 0)$ and $(0, b)$ respectively.



The slope of the line AB is

$$m = \frac{b-0}{0-a} = -\frac{b}{a}$$

The equation of straight line passing through (a, 0) and having slope m is given by

$$y - y_1 = m(x - x_1)$$

That is, $y - 0 = -\frac{b}{a}(x - a)$

or, $\frac{y}{b} = -\frac{x}{a} + 1$

or, $\frac{x}{a} + \frac{y}{b} = 1$

This is called intercept form of a straight line.

Illustration 1

Find the equation of the line through (4, 2) and (–5, –1).

Solution

The equation of the line through two points is

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

So, $y - 2 = \frac{(-1-2)}{(-5-4)}(x - 4)$

or, $y - 2 = \frac{(-3)}{(-9)}(x - 4)$

or, $y - 2 = \frac{1}{3}(x - 4)$

or, $x - 3y + 2 = 0$

Which is the required equation.

1.3 Linear Equations

An equation is a statement that two expressions are equal. A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable. For example, $2x + 3 = 5, x + 2y = 6,$ and $ax + by + cz = d$ are linear equations consisting of one variable, two variables, and three variables.

Solving the equation consists of determining which values of the variables make the equality true. An unknown in the given equation whose value has to be found is known as variable and the value of this variable which satisfies the equality is known as solution. For example, x is the variable in the equation $x + 2 = 5$. In this equation, if we put $x = 3$ then the equation will be satisfied. So $x = 3$ is the root or solution of the equation.

A first-degree or linear equation in one variable x has the form $ax + b = 0$, where a and b are real numbers and $a \neq 0$. For example, $2x + 5 = 11$ is a linear equation in variable x.

Theorem: If a ≠ 0, then the equation ax + b = 0 has precisely one solution, x = – b/a. For example, the solution of equation 2x – 7 = 0 is x = 7/2.

Illustration 2

Solve the equation: $\frac{2x}{x-3} = 4 + \frac{6}{x-3}$.

Solution

We have,

$$\frac{2x}{x-3} = 4 + \frac{6}{x-3}$$

or, $\frac{2x}{x-3} - \frac{6}{x-3} = 4$

or, $\frac{2x-6}{x-3} = 4$

or, $4x - 12 = 2x - 6$

or, $4x - 2x = 12 - 6$

or, $2x = 6$

or, $x = \frac{6}{2} = 3$

1.4 Linear Inequalities

Inequalities express the condition that two quantities are not equal. For example, we write 5 < 8 because 5 is to the left of 8 on the real number line. We may also say that 8 is greater than 5 (written as 8 > 5). If the number *a* is less than or equal to another number *b* then it is written as $a \leq b$. We write $x \geq 5$ to indicate that *x* is greater than or equal to 5. Similarly, if *x* represents the number of drugs a pharmaceutical company produces and sells, if at most 10 different drugs can be produced in a month then this can be written mathematically as $x \leq 10$.

Solving an inequality means finding its solution set, and two inequalities are equivalent if they have the same solution set. As with equations, we find the solutions to inequalities by finding equivalent inequalities from which the solutions can be easily seen. We use the following properties to reduce an inequality to a simple equivalent inequality.

Inequality Properties

An equivalent inequality will result, and the sense or direction will remain the same if each side of the original inequality

1. has the same real number added to or subtracted from it.
2. is multiplied or divided by the same positive number.

An equivalent inequality will result, and the sense or direction will reverse if each side of the original inequality

3. is multiplied or divided by the same negative number.

These properties can be explained as follows:

Let a, b, and c are real numbers.

1. If $a > b$ then $a + c > b + c$ and $a - c > b - c$.
2. If $a > b$ and c is a positive real number, then $ac > bc$ and $a/c > b/c$.
3. If $a > b$ and c is a negative real number, then $ac < bc$ and $a/c < b/c$.

Note: Multiplication by 0 and division by 0 are not permitted.

Thus, we can perform the same operations on inequalities that we perform on equations, with the exception that the sense of the inequality reverses if we multiply or divide both sides by a negative number. Otherwise, the sense of the inequality does not change.

Linear Inequality in One Variables

An inequality is a statement that one quantity is greater than (or less than) another quantity. The inequality $2x - 3 < x + 7$ is an example of first-degree (linear) inequality in one variable that states that the left member is less than the right member. Certain values of the variable will satisfy the inequality. These values form the solution set of the inequality. For example, if we start with the following statement

$$2x - 3 < 5x + 9$$

Adding 3 on both sides, we get

$$2x < 5x + 12$$

Subtracting 5x from both sides, we get

$$-3x < 12$$

Dividing both sides by -3, we get

$$x > -4$$

Thus, $x > -4$ is the solution of the given inequality.

Illustration 3

Solve the inequality $\frac{4(x-3)}{2} \geq x + 5$.

Solution

We have $\frac{4(x-3)}{2} \geq x + 5$

or $4(x-3) \geq 2(x+5)$

or $4x - 12 \geq 2x + 10$

or $4x - 2x \geq 10 + 12$

or $2x \geq 22$

or $x \geq 11$

Thus, $x \geq 11$ is the solution of the given inequality.

Exercise for Reader

1. Find the equation of the line passing through the point (2, 3) and having slope –1/2.
2. Find an equation of the line with slope –2/3 and having y-intercept 3.
3. Find the equation of the line through (–1, 2) and (3, –4)
4. Find the equation of the line passing through (–8, –1) and making equal intercepts on the coordinate axes.
5. Solve the following equations.

   a) $2(x-7)=5(x+3)-x$

   b) $\frac{5x}{2}-4=\frac{2x-7}{6}$

6. Solve the following inequalities.

   a) $\frac{2x}{3}>4-x$

   b) $\frac{4x}{5}-\frac{1}{6}<x-\frac{2(x+1)}{3}$