Course name: ***Algorithm of calculating methods***
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Lecture 13: ***Simplex method for solving the problem of linear programming.***

Fan nomi: ***Hisoblash usullarini algoritmlash***
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13-mavzu: ***Chiziqli dasturlash masalalarini yechishning Simpleks usuli***

13-mavzu. Chiziqli dasturlash masalalarini yechishning Simpleks usuli

Reja:

1. Chiziqli dasturlash masalalarini yechish usullari
2. Simpleks usuli

Tayanch iboralar:

Chiziqli programmalashtirish masalalari, taqribiy yechim, grafik usul, Simpleks usuli.

O'zaro ikki yoqlama simpleks usul

Dastlabki masalaning yechimidan, unga nisbatan ikki yoqlama masalaning yoki ikki yoqlama masalaning yechimidan dastlabki masalaning yechimini keltirib chiqarishga imkon beradigai simpleks usul *o'zaro ikki yoqlama simpleks usul* deyiladi. Bu usul o'zaro ikki yoqlama masalaning asosiy teoremasiga asoslangandir. O'zaro ikki yoqlama simpleks usulning asosiy mazmuni quyidagidan iborat: bizga quyidagi dastlabki

$$Z_{\min} + c'_{m+1}x_{m+1} + c'_{m+2}x_{m+2} + \ldots\ldots + c'_j x_j + \ldots\ldots c'_n x_n = c'_0 . \quad (6)$$

$$\left.\begin{aligned} &\sum_{j=1}^{n} \alpha_{ij} x_j \ge b_i, i = \overline{1,m} \\ &x_j \ge 0, j = \overline{1,n} \end{aligned}\right\} . \quad (7)$$

va unga nisbatan ikki yoqlama

$$F_{\max} = \sum_{j=1}^{m} b_i y_i \quad (8)$$

$$\left.\begin{aligned} &\sum_{i=1}^{m} \alpha_{ij} y_j \le c_j, j = \overline{1,n} \\ &y_i \ge 0, i = \overline{1,m} \end{aligned}\right\} \quad (9)$$

masalalar berilgan bo'lsin. Dastlabki (9.1)-(9.2) va unga nisbatan ikki yoqlama bo'lgan (9.3)-(9.4) masalaga simpleks usulni qo'llash uchun cheklanish shartlari bazis noma'lumlarga nisbatan yechilgan, ya'ni

$$x_{n+1} = -b_1 + \sum_{j=1}^{n} \alpha_{ij} x_j, b_i \le 0 \quad (10)$$

$$y_{m+1} = -\sum_{i=1}^{m} \alpha_{ij} y_i + c_j, c_j \ge 0 \quad (11)$$

ko'rinishda bo'lishi kerak. Bu yerda (10), (11) tenglamalar sistemasi (7) va (9) tengsizliklar sistemasidan qo'shimcha

$$x_{n+i} \geqslant 0, \ i = \overline{1, m}; \ y_{m+j} \geqslant 0, \ j = \overline{1, n}$$

noma'lumlarni kiritish natijasida kelib chiqadi. (10) va (11) da x_{p+1}, x_{p+g}, . . ., x_{n+m}noma'lumlar berilgan masala uchun bazisdir. x_1, x_2, . . ., x_n lar ozod noma'lumlardir, y_{m+1}, y_{m+2}, . . ., y_{m+n} noma'lumlar esa ikki yoqlama masala uchun bazis, u_1, u_2, . . ., $u_m$lar esa ozod noma'lumlardir.

O'zaro ikki yoqlama masalaning asosiy teoremasiga asosan $\min Z = \max F$ bo'lgani uchun yuqoridagi masalalarning birortasining optimal yechimini topsak, ikkinchisining ham optimal yechimini topgan bo'lamiz.

Buning uchun, berilgan masaladagi bazis noma'lumlar bilan ikki yoqlama masaladagi ozod noma'lumlar va berilgan masaladagi ozod noma'lumlar bilan ikki yoqlama masaladagi bazis noma'lumlar o'rtasida o'zaro bir qiymatli moslik o'rnatish kifoyadir, ya'ni:

$$x_{n+1}\, x_{n+2} \ldots\ldots\ldots\ldots x_{n+m}\quad x_1 \ldots\ldots\ldots\ldots x_n$$

(12)

$$y_1\quad y_2 \ldots\ldots\ldots\ldots y_m\quad y_{m+1} \ldots\ldots\ldots\ldots y_{m+n}$$

Agar berilgan masalaning optimal yechimi (0, 0, .. ., $x_{n+1}^{x}_{n+1}$,•••, x_{n+m}bo'lsa, unga ikki yoqlama bo'lgan masalaning optimal yechimi $\{y_1, y_2, \ldots, y_m, 0, 0, \ldots, 0\}$ bo'lib, y_1 x_{n+1} ning oldidagi koeffytsiyentga, ya'ni $y_1 = c_{n+1}$ ga; y_2 esa x_{n+2} ning oldidagi koeffitsiyentga, ya'ni $y_2 = c_{n+2}$ ga; y_m esa x_{n+m} ning oldidagi koeffitsiyentga, ya'ni $y_m = C_{n+m}$ga tengdir.

Misol. 1. Quyidagi masalaga ikki yoqlama masala tuzilsin va ularning yechimi o'zaro ikki yoqlama simpleks usulda topilsin.

$$\begin{aligned}
&Z_{\min} = x_2 - x_4 - 3x_5\\
&x_1 + 2x_2 + x_4 + x_5 = 1,\\
&-4x_2 + x_3 + 2x_4 - x_5 = 1,\\
&3x_2 + x_5 + x_6 = 5, \quad x_j \ge 0, j = \overline{1,6}
\end{aligned}$$

Yechish. Bizga yuqoridan ma'lum bo'lgan dastlabki masalaga ikki yoqlama masala quyidagichadir:

$$\begin{aligned}
&F_{\max} = y_1 + 2y_2 + 5y_3\\
&2y_1 - 4y_2 + 3y_3 \le 1,\\
&-y_2 + 2y_2 \le 1,\\
&y_1 - y_2 + y_3 \le -3, \quad y_j \le 0, j = \overline{1,3}
\end{aligned}$$

yoki

$$\begin{aligned}
&F_{\max} = y_1 + 2y_2 + 5y_3\\
&2y_1 - 4y_2 + 3y_3 + y_4 = 1,\\
&-y_2 + 2y_2 + y_5 = 1,\\
&y_1 - y_2 + y_3 + y_6 = -3, \quad y_j \le 0, j = \overline{1,3}
\end{aligned}$$

Dasglabki masalada x_1, x_3 ва x_6 noma'lumlar bazis, x_2, $x_4$va x_5 noma'lumlar ozod noma'lumlardir. Ikki yoqlama masalada esa y_4, y_5 va y_6

noma’lumlar bazis, y_1, y_2 va y_3 noma’lumlar ozod noma’lumlardir. Bu misol uchun (12] munosabat quyidagicha bo‘ladi:

$$\begin{matrix} x_1 & x_3 & x_6 \\ \updownarrow & \updownarrow & \updownarrow \\ y_1 & y_2 & y_3 \end{matrix} \qquad \qquad \begin{matrix} x_2 & x_4 & x_5 \\ \updownarrow & \updownarrow & \updownarrow \\ y_4 & y_5 & y_6 \end{matrix}$$

Dastlabki masalaning cheklanish shartlarida ozod hadlar hammasi musbat bo‘lgani uchun simpleks usulini qo‘llash osondir. Dastlabki masalaga mos keluvchi quyidagi jadvalni tuzamiz:

6-jadval

Bazis noma’lumlar	Ozodhadlar	X_1	X_2	X_3	X_4	X_5	X_6
X_1	1	1	2	0	-1	(1)	0
X_2	2	0	-4	1	2	-1	0
X_3	5	0	3	0	0	1	1
Z	0	o	-'	0	1	3	0

Simpleks jadvallarning biridan ikkinchisiga o‘tib, quyidagi jadvallarni tuzamnz (7-9- jadvallar):

7-jadval

Bazis noma’lumlar	Ozodhadlar	X_1	X_2	X_3	X_4	X_5	X_6
X_1	1	1	2	0	-1	1	0
X_2	3	1	-2	1	1	0	0
X_3	4	-1	1	0	1	0	1
Z	-3	-3	-7	0	4	0	0

8-jadval

Bazis noma’lumlar	Ozodhadlar	X_1	X_2	X_3	X_4	X_5	X_6
X_5	4	2	0	1	0	1	0
X_2	3	1	-2	1	1	0	0
X_3	1	-2	3	-1	0	0	1
Z	-15	-7	1	4	0	0	0

9-jadval

Bazis noma’lumlar	Ozodhadlar	X_1	X_2	X_3	X_4	X_5	X_6
X_1	4	2	0	1	0	1	0
X_2	11/3	-1/3	0	1/3	1	0	2/3
X_3	1/3	-2/3	1	-1/3	0	0	1/3
Z	-46/3	-19/3	0	-11/3	0	0	-2/3

9-jadvalning oxirgi satrida musbat element mavjud emas. Demak, topilgan (0; 1/3; 0; 11/3; 4,0) yechim dastlabki masalaning manfiy bo‘lmagan optimal yechimi bo‘ladi. Bu yechimda maqsad funksiya quyidagicha bo‘lib:

$$Z = \frac{46}{3} - \frac{19}{3}x_1 - \frac{11}{3}x_3 - \frac{1}{3}x_6 \qquad (13)$$

uning qiymati $Z = -\frac{46}{3}$ ga teng. U holda

$C_1 = -\frac{19}{3}, \quad C_3 = -\frac{11}{3}, \ C_6 = -\frac{1}{3}, \ C_2 = C_4 = C_5 = 0.$

Yuqoridagi o‘zaro bir qiymatli moslikka asosan, $y_1 \leq <0$, $y_2 \leq <0$ va $y_3 \leq <0$ ni nazarda tutsak:

$$y_1 = -\frac{19}{3}, \ \ y_2 = -\frac{11}{3}, \ \ y_3 = -\frac{1}{3}, \ \ y_4 = y_5 = y_6 = 0$$

bo‘ladi. Demak, ikki yoqlama masalaning optimal yechimi

$$\left\{-\frac{19}{3}; -\frac{11}{3}, \ -\frac{1}{3}; \ 0; \ 0; \ 0\right.$$

bo‘lib,

$$\max F = \min\{y_1 + 2y_2 + 5y_3\} = -\frac{46}{3}. \qquad (14)$$

Shunday qilib, asosiy teoremaning sharti quyidagicha bajariladi:

$\min Z = \max F = -\frac{46}{3}.$

Misol. 2. Quyidagi

$$Z_{\min} = x_1 + 2x_2 + 3x_3 \qquad (15)$$

$$\left.\begin{aligned} &2x_1 + 2x_2 - x_3 \geq 2, \\ &x_1 - x_2 - 4x_3 \leq -3, \\ &x_1 + x_2 - 2x_3 \geq 6, \\ &2x_1 + x_2 - 2x_3 \geq 3, \\ &x_j \geq 0, j = \overline{1,3} \end{aligned}\right\} \qquad (16)$$

masalaga ikki yoqlama masala tuzilsin va ularning yechimi o‘zaro ikki yoqlama simpleks usul bilan topilsin.

Yechish. Bu masalani yechish uchun avval cheklanish shartlari (17) dagi barcha tengsizliklarning ishorasini ≥ ko‘rinishga keltiramiz. Buning uchun (17) ning ikkinchisini „-1" ga ko‘paytirib, quyidagiga ega bo‘lamiz:

$$\left.\begin{aligned} &2x_1 + 2x_2 - x_3 \geq 2, \\ &-x_1 + x_2 + 4x_3 \geq 3, \\ &x_1 + x_2 - 2x_3 \geq 6, \\ &2x_1 + x_2 - 2x_3 \geq 3 \end{aligned}\right\} \qquad (17)$$

yoki tenglama ko‘rinishida quyidagicha bo‘ladi:

$$\left.\begin{aligned} x_4 - 2x_1 - 2x_2 + x_3 = 2, \\ x_5 + x_1 - x_2 - 4x_3 = 3, \\ x_6 - x_1 - x_2 + 2x_3 = 6, \\ x_7 - 2x_1 - x_2 + 2x_3 = 3 \end{aligned}\right\} \quad (18)$$

(15)- (16) da ikki yoqlama masala quyidagicha bo‘ladi:

$$F_{\max} = 2y_1 + 3y_2 + 6x_3 + 3y_4$$

$$\left.\begin{aligned} & 2y_1 - y_2 + y_3 + 2y_4 \le 1, \\ & 2y_1 + y_2 + y_3 + y_4 \le 2, \\ & -y_1 + 4y_2 - 2y_3 - 2y_4 \le 3, \\ & y_i \ge 0, i = \overline{1,4} \end{aligned}\right\} \quad (19)$$

yoki

$$\left.\begin{aligned} & 2y_1 - y_2 + y_3 + 2y_4 + y_5 = 1, \\ & 2y_1 + y_2 + y_3 + y_4 + y_6 = 2, \\ & -y_1 + 4y_2 - 2y_3 - 2y_4 + y_7 = 3, \\ & y_i \ge 0, i = \overline{1,7} \end{aligned}\right\} \quad (20)$$

Dastlabki masaladagi x_4, x_5, x_6, x_7 bazis noma’lumlar bilan ikki yoqlama masaladagi y_1, y_2, y_3 y_4 ozod noma’lumlar va berilgan masaladagi x_1, x_2, x_3 ozod noma’lumlar bilan ikki yoqlama masaladagi y_5, y_6, y_7 bazis noma’lumlar o‘rtasida quyidagi

$$\begin{matrix} x_4 & x_5 & x_6 & x_7 & \quad & x_1 & x_2 & x_3 \\ \\ y_1 & y_2 & y_3 & y_4 & \quad & y_5 & y_6 & y_7 \end{matrix} \quad (21)$$

bir qiymatli moslikni o‘rnatamiz va bevosita birinchisining yechimlaridan ikkinchisiniig yechimlarini keltirib chiqaramiz. Endi (18) va (20) shartlarga nazar tashlasak, (20) da hamma ozod hadlar musbat bo‘lgani uchun, ikki yoqlama (19)-(20) masalani simpleks usuli bilan yechish qulayroqdir.

Shuning uchun, ikki yoqlama masalani simpleks usuli bilan yechamiz. Bumasalaning yechimi 6-jadvalda keltirilgan (21) munosabatga asosan berilgan nasalaning optimal yechimi

$$\{x_1, \ x_2, \ x_3, \ 0, \ 0, \ 0, \ 0\}$$

bo‘lib, $x_1 = c_5$. $x_2 = c_6$, $x_3 = c_7$ bo‘lishi kerak. Haqiqatan ham 9-jadvaldan (19)-(20) masalaning optimal yechimi

$$\left\{0; \ \frac{1}{2}; \ \frac{3}{2}; \ 0; \ 0; \ 0; \ 2\right\}$$

bo‘lib, bu yechimda

$$F_{\max} = \frac{21}{2} - (10y_1 + \frac{9}{2}y_4 + \frac{3}{2}y_5 + \frac{9}{2}y_6) \quad (22)$$

ning qiymati $F = \frac{21}{2}$ ga tengdir.

10-jadval

Bazis noma’lumlar	Ozodhadlar	y_1	y_2	y_3	y_4	y_5	y_6	y_7
y_5	1	2	-1	1	2	1	0	0
y_6	2	2	1	1	-1	0	1	0
y_7	3	-1	4	-2	-1	0	0	1
Z	0	-2	-3	-6	-3	0	0	0

11-jadval

Bazis noma’lumlar	Ozodhadlar	y_1	y_2	y_3	y_4	y_5	y_6	y_7
y_3	1	2	-1	1	2	1	0	0
y_6	1	0	2	0	-1	-1	1	0
y_7	5	3	2	0	2	2	0	1
Z	6	10	-9	0	6	0	0	0

12-jadval

Bazis noma’lumlar	Ozodhadlar	y_1	y_2	y_3	y_4	y_5	y_6	y_7
y_5	3/2	-2	0	1	3/2	1/2	1/2	0
y_2	½	0	1	0	1/2	1/2	1/2	0
y_7	2	3	0	0	4	5	3	1
Z	21/2	10	0	0	9/2	3/2	9/2	0

9.7-jadval dan noma’lumlarning qiymatlarini topamiz

$$x_1 = c_5 = \frac{3}{2}, \quad x_2 = c_6 = \frac{9}{2}, \quad x_8 = c_7 = 0$$

Berilgan masalaning optimal yechimini topamiz. Bu yechiming qiymati quyidagicha:

$$Z_{min} = \frac{3}{2} + \frac{18}{2} = \frac{21}{2}.$$

Demak, asosiy teoremaning quyidagi sharti bajariladi

$$\min Z = \max F = \frac{21}{2}$$

Bu esa masalaning to‘gri yechilganini bildiradi.

Takrorlash uchun savollar:

1. Chiziqli dasturlash masalalarini yechishning qanday usullarini bilasiz?
2. Simpleks usuli nima?
3. Taqribiy yechim qanday topiladi?
4. Grafik usul qanday?