CONCEPT OF TIME AND VALUE OFMONEY

Simple and Compound interest

What is the future value of shs 10,000 invested today to earn an interest of 12% per annum interest payable for 10 years and is compounded;

a. Annually (3 marks)
b. Semi-quarterly (2 marks)
c. Quarterly (3 marks)
d. Monthly (2 marks)

Solving

(i) Annually $FV = PV \times (1 + r)^n$

$= 10,000 \text{ x } (1.12)^{10}$

$= 310,584.8$

(ii) Semi-annually

$FV = PV \text{ X } (1 + \frac{r}{2})^{10(2)}$

$= 10,000\ (1+\frac{0.12}{2})^{20}$

$= 320,713.5$

(iii) Quarterly $= FV= PV\ (1 + \frac{r}{4})^n m$

$= 100,000\ (1 + \frac{0.12}{4})^{10(4)}$

$= 326,203.77$

(iv) Monthly

$FV = PV\left(1 + \frac{r}{12}\right)^{n m}$

$= (10{,}000\left(1+\frac{0.12}{12}\right)^{12}$

= 330,038.6

QUESTION 2 (c)

Effective Annual Rate

This is the interest rate expressed as if it were compounded once per year. The actual rate of interest earned (paid) after adjusting the nominal rate for factors such as the number of compounding periods per year. The effective annual interest rate is the interest rate compounded annually but provides the same annual interest as the nominal rate does when compounded *m* times per year.

Example 1

For example, suppose you are offered 12 percent compounded monthly. In this case, the interest is compounded 12 times a year; so *m* is 12. You can calculate the effective rate as:

Solving

EAR = [1 + (Quoted rate)/*m*]м - 1

$= [1 + .12/12]^{12} - 1$

$= 1.01^{12} - 1$

= 1.126825 – 1

=12.6825%

Example 2

A bank is offering 12 percent compounded quarterly. If you put $ 100 in an account, how much will you have at the end of one year? What's the EAR? How much will you have at the end of two years?

Solving

The bank is effectively offering 12%/4 = 3% every quarter. If you invest $ 100 for four periods at 3 percent per period, the future value is:

Future value = $ 100 x $(1.03)^4$

= $ 100 x 1.1255

= $ 112.55

The EAR is 12.55 percent [$ 100 x (1 + .1255) = $ 112.55].

We can determine what you would have at the end of two years in two different ways. One way is to recognize that two years is the same as eight quarters. At 3 percent per quarter, after eight quarters, you would have:

$ 100 x $(1.03)^8$ = $ 100 x 1.2668 = $ 126.68

Alternatively, we could determine the value after two years by using an EAR of 12.55 percent; so after two years you would have:

$ 100 x $(1.1255)^2$ = $ 100 x 1.2688 = $ 126.68

Thus, the two calculations produce the same answer. This illustrates an important point. Anytime we do a present or future value calculation, the rate we must be an actual or effective rate. In this case, the actual rate is 3 percent per quarter. The effective annual rate is 12.55 percent. It doesn't matter which one we use once we know the EAR.

Annuity

An annuity is a series of consecutive payment or receipts of equal amount over a defined period of time.

Usually, the receipts or payment are assumed to occur at the end of the year.

Can also be defined as a series of equal amount payment for a specified number of years

It is a level stream of cash flow for a fixed period of time.

For example, a loan repayment plan calls for the borrower to repay the loan by making a series of equal payment for some length of time.

A series of constant or level cash flows that occurs at the end of each period is called an ordinary annuity.

Compound Annuities: (ANNUITY FUTURE VALUE)

It involves depositing or investing an equal sum of money at the end of each year for a certain number of years and allowing it to grow

Example 1

Assume you want to deposit $500 for college education at the end of each year for the next 5 years in a bank. The money will earn 6 percent interest. How much money will be there at the end of 5^{th} year.

$$FV5 = \frac{PMT\ [1+r]^n - 1}{r} \qquad \text{(ANNUITY FUTURE VALUE)}$$

FV5=$500 X(FVIFA)

=500x5.637

$2818.5

Example 2

How much must we deposit in an 8 percent saving accounts at the end of each year to accumulates $5000 at the end of 10 years.

$$FV = PMT \frac{[1+r]^n - 1}{r}$$

5000=PMTX14.4866

$$PMT = \frac{5000}{14.4866}$$

$345.15

Example 3

An investor deposits sh. 1000 at the end of each year for four years in an account earning interest at the rate of 10% per annum.

What is the value at the end of the fourth year?

Future value of an annuity is given by:

$$FV = \left[\frac{(1+r)^n - 1}{r}\right]$$

A= Periodic annuity amount

$(1+ r)^n$ = Future value interest factor of annuity (FV)

r = Discount rate/interest rate

Annual annuity Amt (A) = sh. 1000

Number of years (n) = 4 years

$$\text{Therefore } FV = 1000 \left(\frac{1.1^4 - 1}{0.1}\right)$$

FV= 1000 X 4.6410

= 4641

Example 4

What would an investor have to deposit at the end of each year at an interest rate of 6% if he wishes to accumulate sh. 10,000 in 5 years?

Annual Annuity Amount A =?

Number of years = 5

Interest r = 6%

$$10{,}000 = A\ \frac{[(1.06)^5 - 1]}{0.06}$$

A = 1,774

If an annuity is made at the beginning rather than the end of the period, it is referred to as annuity due.

The future value of an annuity due is related to a future value of a normal annuity by the expression:

FV annuity due = FV normal annuity * (+ r)

Example5

Suppose you plan to contribute $ 2,000 every year into a retirement account paying 8 percent. If you retire in 30 years, how much will you have?

Solution

Future value = Annuity present value x $(1.08)^{30}$

$$= \$\ FV = \frac{PMT\ [1+r]^n - 1}{r} x(1.08)^n$$

Annuity present value = $ 2000 x 11.32832

= $ 22, 515.57

The future value of this amount in 30 years is:

22,515.57 x1.08^30

$226,566.4

Present value for annuity cash flow

Pension payment, insurance obligation and the interest owed on bond all involve annuities.

To compare these three types of investments, we need to know the present value of each

The present value of an annuity is given by the expression:

$$PV = A \left[\frac{1- (1/ (1 + r)^{n}}{r}\right]$$

Example 1

Suppose you are receiving $500 at the end of each year for the next 5 years. The discount rate is 6 percent. What is the worth of this investment today?

$$PV = A \left[\frac{1- (1/ (1 + r)^{n}}{r}\right]$$

=500x4.212

$2106

Example 2

What is the present value of sh. 10,000 to be received at the end of each year for 5 years at a rate of interest of 10%?

$$\text{Annuity Present value factor} = PMT \left[\frac{1 - (1/(1 + r)t}{r}\right]$$

$$= 10,000 \left(\frac{1- 1.1^{5}}{0.1}\right)$$

= 10,000 x 3.7908

= 37,908

Example 3

After carefully going over your budget, you have determined you can afford to pay $ 632 per month towards a new sports car. You call up your local bank and find out that the rate is 1 percent per month for 48 months. How much can you borrow?

$$\text{Annuity present value} = PMT \times \left[\frac{\text{present value factor})}{r}\right]$$

$$\text{Annuity Present value factor} = PMT \left[\frac{1 - (1/(1 + r)t}{r}\right]$$

Therefore present value = $ 632 x 37.9740

= $ 24,000

Therefore, $ 24000 is what you can afford to borrow and pay.

Amortization of loan

An important use present value concept is in determining the payment required for an installment-type loan. The distinguishing feature of this loan is that it is repaid in equal period payment that includes both interest and principal. This payment can be made monthly, quatertely, semi annually or annually. Installment payments are prevalent in mortgages loans, auto loans, consumer loans and certain business loans.

Finding the payment/ amortizing a loan

Example one

Suppose you wished to start up a new business that specializes in the latest of health food trends, frozen Yak milk. To produce and market your product, the Yankee Dandy, you need to borrow $ 100,000. Because it strikes you as unlikely that this particular food will be long lived, you propose to pay off the loan quickly by making five equal annual payments. If the interest rate is 18%, what will be the payment?

Annuity present value = $ 100,000 = PMTx [1 - present value factor]

100,000 = PMT X (1 - $\frac{1/(1.18)^5}{0.18}$

PMTX (3.1272)

PMT= $ $\frac{100,000}{3.1272}$ = $ 31,977.49

Therefore, you will make a payment of $ 32,000 each.

Example two

You borrow $10,000 at 14 percent compound annual interest for four years. The loan is repayable in four equal annual installments payable at the end of each year.

a. What is the annual payment that will completely *amortize* the loan over four years? (You may wish to round to the nearest dollar.)

b. Of each equal payment, what is the amount of interest? The amount of loan principal?

Solving

Annuity Present value factor = PMT [1 - (1/(1 + r)t] / r

USE TABLE FOR ANNUITY

a. PV_0 = $10,000 = R($PVIFA_{14\%,4}$) = R(2.914)

Therefore R = $10,000/ 2.914 = $3,432 (to the nearest dollar).

(1)	(2)	(3)	(4)	
(END OF	INSTALLMENT	ANNUAL	PRINCIPAL	PRINCIPAL AMOUNT YEAR END
PAYMENT	INTEREST	PAYMENT	OWING AT YEAR END	
		(4)t_{-1} X 0.14	(1) – (2)	(4) t_{-1} - 3)
0	-	-	-	$10,000
1	$3,432	$1,400	$2,032	7,968
2	3,432	1,116	2,316	5,652
3	3,432	791	2,641	3,011
4	3,432	421	3,011	0
	$13,728	$3,728	$10,000	

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3	3,432	791	2,641	3,011
4	3,432	421	3,011	0
	$13,728	$3,728	$10,000	

Example 3

You ran a little short on your spring break vacation, so you can put $ 1000 on your credit card. You can only afford to make the minimum payment of $ 20 per month. The interest rate on the credit card is 1.5 percent per month.

How long will you need to pay off the $ 1000?

$$\$ 1000 = \$ 20 \times \frac{1 - \text{present value factor}}{0.015}$$

$$\left(\frac{\$ 1000}{20}\right) \times 0.015 = 1 - \text{present value factor}$$

Present value factor = 0.25 = 1/ (1+r) t

(1.015) t = 1/0.25 = 4

This boils down to asking this question

How long does it take for your money to quadruple at 1.5 percent per month?

$(1.015)^{93} = 3.99 = 4$

It will take you about 93/12 = 7.75 years at this rate.

SUMMARY OF TIME VALUE OF MONEY EQUITION

Calculation	**Equation**
Future value of single payment	$FV=PV\ (1+r)^{n}$
Present value	$PV=FV\left(\frac{1}{(1+r)^{n}}\right)$
Future value for an annuity	FV of an annuity= $\frac{PMT\ (1+r)^{n}}{r}$
Present value of annuity	PV of an annuity=PMT $\left(\frac{1-(1+r)^{-n}}{R}\right)$
Future value of an annuity due	FV (annuity due) =future value of an annuity x (1+r)
Present value of an annuity due	PV(annuity due)=present value of an annuity x (1+r)

There are some characteristics that should help you to identify and solve the various types of annuity problems:

1. Present value of an ordinary annuity – cash flows occur at the *end* of each period, and *present value is calculated as of one period before the first cash flow.*

2. Present value of an annuity due – cash flows occur at the *beginning* of each period, and *present value is calculated as of the first cash flow.*

3. Future value of an ordinary annuity – cash flows occur at the end of each period, and future value is calculated as of the last cash flow.

4. Future value of an annuity due – cash flows occur at the beginning of each period, and future value is calculated as of one period after the last cash flow.

Practice questions

1. Your company proposes to buy an asset for $335.This investment is very safe and will be sold in three years time for $400.You knows that you could invest the $335 elsewhere at 10% with very little risk. What do you think of the proposed investment?
   $335 x(1+r)^t=

2. You are considering a one year investment. If you put up $1250,you will get back $1350.What is the rate this investment is paying
   $1250=$1350/(1+r)^t

3. You estimate that you will need about $80000 to send your child to college in 8 years .You have about $35000 now. If you can earn 20 percent per year, will you make it? At what rate will you just reach your goal?
   - FV=$35000 x(1.2)^8
   - FV=$35000 x(1+r)^8=$80000
   - (1+r)=$80000/35000=2.2857 use table

4. You are offered an investment that will pay you $200 in one year, $400 the next year,$600 the next year, and $800 at the end of last year. You can earn 12 percent on very similar investment. What is the most will you be willing to pay?
   $200 x1/1.12^1=178.57
   $400 x1/1.12^2=318.88
   $600 x1/1.12^3=427.07
   $800 x1/1.12^4=508.41
   $1432.93
5. An insurance company offers to pay you $1000 per year for 10 years if you pay $6710 up front. What rates is implicit in this 10 years annuity.
   $6710=$1000 x(1-present annuity value factor)/r

6. Suppose you plan to contribute $2000 every year into retirement account paying 8 percent. If you retire in 30 years, how much will you have?
   Annuity present value=$2000 x(1-1/1.08^30)/.08

7. Assume you want to deposit $500 for college education at the end of each year for the next 5 years in a bank. The money will earn 6 percent interest. How much money will be there at the end of 5^{th} year?
8. How much must we deposit in an 8 percent saving accounts at the end of each year to accumulate $5000 at the end of 10 years.

9. An investor deposits sh. 1000 at the end of each year for four years in an account earning interest at the rate of 10% per annum.
   What is the value at the end of the fourth year?

10. You borrow $10,000 at 14 percent compound annual interest for four years. The loan is repayable in four equal annual installments payable at the end of each year.
    c. What is the annual payment that will completely *amortize* the loan over four years? (You may wish to round to the nearest dollar.)

    d. Of each equal payment, what is the amount of interest? The amount of loan principal?