Total lecture of 2 hr 30 minutes

(Divided into 3 lectures of 50 minutes each)

Lecture – 1(50 minutes)

Set Theory:

a) Definition of a set
b) Set representation
c) Some special sets (Null, Singleton, Universal)
d) Class works

Definition of a set:

A well-defined collection of objects is said to be a set. A set may be thought as well defined list, collection or class of objects. The objects may be material or conceptual. Anything contained in a set is called an element or member of the set. Generally, the sets are denoted by capital letters.

Some examples of sets are given below:

1. The countries of Asia.
2. The vowels of the English alphabets.
3. The natural numbers.
4. The solar system.

Notations:

Sets are usually denoted by capital letters A, B, C, D,..., X, Y, Z and their elements by small letters a, b, c, x, y, z, The symbol $\in$ is used to indicate the set membership and read as "belongs to" or "is an element of" or "is a member of". The symbol $\notin$ is used to indicate the set non-membership and read as "does not belong to" or "is not an element of or "is not a member of". The symbol $\Rightarrow$ is use to show the consequence of the previous step and read as "implies" The symbol $\Leftrightarrow$ is used to show that the previous step implies the second step and also the second step implies the previous step and is read as "if and only if'. it is also denoted by "iff".

Set representation

The following are the various methods by which the sets can be described or specified.

i) Listing or Tabulation method.
ii) Description method.
iii) Rule method or Set builder method.

<u>i) Listing, or Tabulation method.</u>

In listing or tabulation method, the elements are listed, separated by commas and enclosed in braces { }.

Examples:

A= {a, e, i, o, u}

B = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

ii) Description method.

In this method, we specify the set by stating some descriptive phrase or by enclosing in braces a descriptive phrase.

Examples:

A = The set of vowels of the English alphabet.

B = {Vowels of the English alphabet}

iii) Rule method or Set builder method.

In this method the set is specified by stating the property which its every element satisfies.

Example:

If A is the set of vowels of English alphabet then in set builder form it is:

A = {x: x is a vowel of English alphabet).

We read this as "A is the set (or collection) of all x such that x is a vowel of English alphabet"

Some special sets:

1. Null or Empty or void sets:

A set which has no element is called the null set or empty set or void set. It is denoted by ϕ

For example:

i) The set of male students of St. Mary's school.

ii) M = {x: x is a man of height 15 metres }

2. Singleton sets:

A set with only one element is said to be a singleton. It is also known as unit set.

For example:

1) A = {9}

ii) H = {x: x is the highest mountain on the earth}

> **Note:** The difference between ϕ and $\{\phi\}$ is that ϕ is a null set and $\{\phi\}$ is a singleton set.

3. Universal Set:

The set of all objects or things under consideration in a discussion is called the universal set or simply the universe. It is usually denoted by U. For example:

i) In the school arithmetic, the universal set is the set of all numbers.
ii) In human population studies, the universal set consists of all people in the world.

Class activities and class works.

1. What do you mean by a set ? When is a collection non defined and when it is well defined? Give examples.

2. Write the following sets in the tabular form:
   i) The set of months in a year according to the Nepali Calendar.
   ii) The set of digits in the Hindu Arabic Numeral System.
   iii) The set of even integers between 11 and 19.
   iv) A = {x : x is a letter of the word "empty"}

3. Write each of the following in the set builder form:
   i) The set of books in a library.
   ii) {a, e, ı, o, u}
   iii) {sight, hearing, smell, taste, touch)
   iv) { 1, 1/2, 1/3, 1/4, 1/5}
4. What do you mean by a null set? How is it represented?
5. What is Singleton set? Give an example.
6. What are the characteristics of a universal set?

Lecture 2 (50 minutes)

a) Relation between sets: Subset, Equal sets, Disjoint sets, Intersecting sets, Equivalent sets.
b) Venn diagrams and set operations: Union, Intersection, Difference, Symmetric difference, Complement

A. Relation between sets:

i) Subset.

If every element of a set is also an element of other, then the first set is said to be a subset of the other. If A is a subset of B then it is denoted by $A \subseteq B$.
Symbolically $A \subseteq B$ iff $x \in A \Rightarrow x \in B$.

Examples:

a) Let A = {1, 2, 3} and B = {1, 2, 3, 4}. Here every element of A is also an element of B. So, $A \subseteq B$.
b) Let A= {a, b, c} and B= {a, b, c}. Here every element of A is also an element of B. So $A \subseteq B$ and every element of B is also an

element of A. So B$\subseteq$ A also.

ii) Proper subset.

If A and B are any two non-empty sets such that every element of A is an element of B but there is at least one element of B which is not of A then A is said to be the proper subset of B. It is denoted by A$\subset$ B.

Example:

Let A= {a, b, c} and B= {a, b, c, d}.

Here every element of A is contained in B but there is an element d of B which is not contained in A. So A is a proper subset of A. That is, A$\subset$B.

Note: If a set A is subset of B but not the proper subset then it is said to be improper. If A= {a, b, c} and B= {a, b, c} then A is an improper subset of B and B is also an improper subset of A.

iii) Equality of two sets.

Two sets with same elements are said to be equal. In other words, if A and B are any tow non-empty sets such that every element of A is an element of B and every element of B is also an element of A, then the two sets A and B are said to be equal.

Symbolically A = B iff A $\subseteq$ B and B $\subseteq$ A.

Example:

Let A= {x, y, z} and B = {y, x, z}

Here all the elements of A are contained in B and all the elements of B are contained in A. So they are equal. That is A = B.

iv) Equivalent sets.

Two sets with same number of elements are said to be equivalent. To be equivalent sets, the elements contained in one set must not be the same as the elements contained in other set but both the sets must contain equal number of elements.

Example:

Let A= {x, y, z} and B = {a, b, c}

Here the number of elements in set A is 3 and the number of elements in set B is also 3. So A is equivalent to B.

v) Disjoint sets

Two or more sets are said to be disjoint if they have no common element.

Example:

Let A= {x, y, z} and B = {p, q, r, s}

Here the given sets A and B contain no common elements, so they are disjoint sets.

vi) Intersecting sets

Two or more sets are said to be intersecting if they have at least one element common.

Example:

Let A= {x, y, z} and B = {a, x, z, s}

Here the given sets A and B both contain the elements x and z. So they have two elements common and they are intersecting sets.

B.Venn diagrams:

A simple diagram to represent the sets, relation between sets and operation on sets, is known as Venn diagram. In Venn diagram, the universal set is represented by the rectangle, a set (a subset of U) by the circle and the elements of the set by the points kept within the circle. We shall now illustrate the operations of union, intersection, complementation and difference by means of Venn diagrams. The interpretation of the diagrams are given on the right.

i. Union

The union of two sets A and B is the set of all elements that belong either to A or to B or to both A and B. We denote the union of A and B by $A \cup B$. Symbolically, it is defined as:

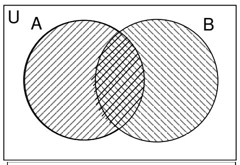
$A \cup B = \{x: x \in A \text{ or } x \in B\}$

Example.-

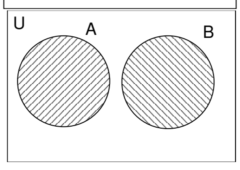
If A = {2, 3, 4, 5} and B = {4, 5, 6, 7}

then $A \cup B$ = {2, 3, 4, 5, 6, 7}

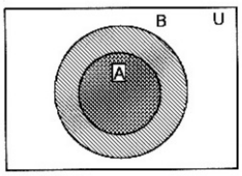
Note: the elements in a set are not repeated.



The total shaded area is $A \cup B$.
Here A and B are intersecting sets.



The shaded area is $A \cup B$.
Here A and B are disjoint.



The total shaded area is $A \cup B$.
Here $A \subset B$.
In this case $A \cup B = B$

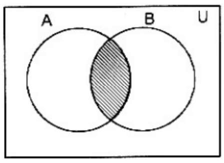
ii) Intersection

The intersection of two sets A and B is the set of all elements which belong to both the sets A and B. We denote the intersection of A and B by A∩B. Symbolically, it is defined as: $A \cap B = \{x : x \in A \text{ and } x \in B\}$

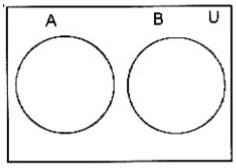
Example:

If A = {2, 3, 4, 5} and B = {4, 5, 6, 7}

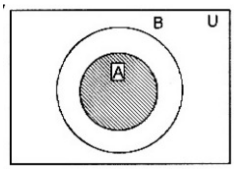
then A∩B = {4, 5}



The shaded portion is
A∩B
Here A meets B.



Here A and B are disjoint.
No shaded portion for A∩B.
Here A∩B = φ.



The shaded portion is
A∩B.
Here A ⊂ B.
In this case A∩B = A

iii. Difference

The difference of sets A and B is the set of all elements of A that do not belong to the set B. We denote the difference of A and B (not B and A) by A - B. We read this as "A difference B".

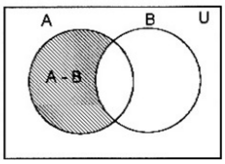
Symbolically, it is defined as:

$A - B = \{x : x \in A, x \notin B\}$

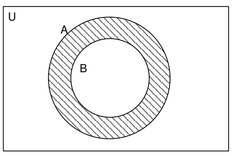
Example:

If A = {2, 3, 4, 5} and B = {4, 5, 6, 7}

then A - B = {2, 3} because 2, 3 are the elements which belong to the set A and do not belong to B.

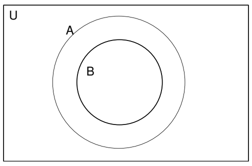


The shaded part is
A - B.
Here two sets are intersecting.

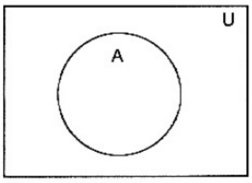


The shaded part is

A - B.
Here $B \subset A$



If $B \subset A$, There will be no element in B - A. So no area is shaded.
Hence B - A = ϕ, for $B \subset A$



No shaded part for the difference A - A.
Hence A - A = ϕ

iv. Complement

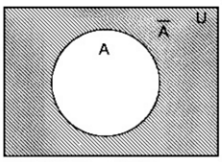
The complement of a set A is the set of all elements in the universal set U that do not belong to A. i. e. complement of a set A is the set U - A. It is denoted by $\overline{A}$, which is read "A bar" or "the complement of A". The symbol A', A^c, C(A) etc. are also used to denote the complement of A.

Symbolically, it is defined as:

$A^c = \{x: x \in U, x \notin A\}$

Fxample:

If U= {1, 2, 3, 4, 5, 6, 7, 8, 9, 0} and
A=(2,3,4,5} then A^c = { 1, 6, 7, 8, 9, 0}



The shaded portion is A^c.
This is complement of A.
Here A^c = U - A

v. Symmetric difference:

The symmetric difference of two sets A and B is the union of A-B and B-A. That is, the symmetric difference of two sets A and B is the union of only A and only B. The symmetric difference of A and B is denoted by $A \Delta B$.

Example:

Let A = { 1, 2, 3, 4}, B = {2, 4, 6, 8}. Here A – B = {1, 3} and B-A = {6, 8}.
So $A \Delta B = (A-B) \cup (B-A)$ = {1, 3, 6, 8}

Class activities and class works:

1. What do you mean by Venn diagram?
2. With an example, explain union, intersection, complement and difference of two sets.

3. Illustrate the following by Venn diagrams:
   i) A = {a, b, c, d} and B = {e, b, c, d}
   ii) A ={a, c, d} and B = {b, e, f}
   iii) A = {a, b, c} and B = {a, b, c, d, e}
4. Let U = {a, e, i, o, u}, A = {i, a}, B = {u}, C = {i, o, u}, find the following.
   i) A'   ii) B'   iii) $A'\cup B'$
5. If U = {a, b, c, d, e, f, g}, A = {a, b, c, d, e)
   B = {a, c, e, g} and C = {b, e, f, g}
   verify that: i) $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$
   iii) $(A\cup B)' = A'\cap B'$.

Lecture-3 (50 minutes)

a) Cardinal number and some formulae.
b) Use of Venn diagrams to explain the formulae.
c) Some examples to use the formulae.

A. (Cardinal number) Number of elements in a set:

The number of elements in a set is known as the cardinal number of that set. If A and B are two sets, then the number of elements in the sets A and B are denoted by n(A) and n(B) respectively. Thus if n(A) = n(B), then the sets are equivalent.

If A, B and C are the subsets of the universal set U then

i) $n(A\cup B) = n(A) + n(B) - n(A\cap B)$.

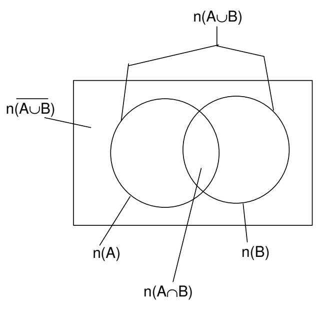
ii) $n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$.

iii) $n(A') = n(U) - n(A)$.

iv) $n(A - B) = n(A) - n(A\cap B)$.

v) If $A\subset B$ then $A\cap B = A$. So, $n(A\cap B) = n(A)$.
[This gives maximum value of $n(A\cap B)$]

vi) If $A\subset B$ then $A\cup B = B$. So, $n(A\cup B) = n(B)$.
[This gives minimum value of $n(A\cup B)$] Examples:



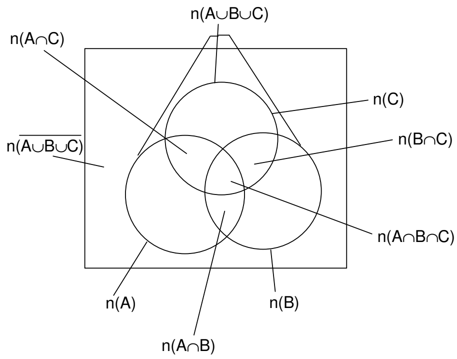
In the above diagram, we see that the $n(A\cup B)$ covers the total area of both circles. $n(A)$ covers one circle and $n(B)$ covers other circle. $n(A\cap B)$ is the area which is in the intersection of sets A and B. So $n(A) + n(B)$ takes $n(A\cap B)$ twice and covers the total area of $n(A)$ and $n(B)$. That is, $n(A) + n(B)$ is more than $n(A\cup B)$ by $n(A\cap B)$.

Hence $n(A\cup B) = n(A) + n(B) – n(A\cap B)$.

Also from the diagram $n(A-B) = n(A) - n(A\cap B)$ and

$n(B-A) = n(B) - n(A\cap B)$

Similarly the following diagrams show the reasons beyond the formulas given above:



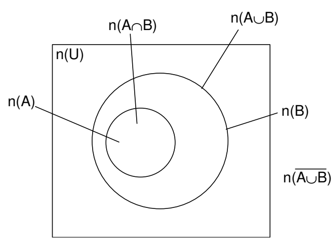
From this diagram we see that

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$.

here, $n(A) + n(B) + n(C)$ takes $n(A\cap B\cap C)$ thrice and

$- n(A\cap B) - n(B\cap C) - n(C\cap A)$ subtracts $n(A\cap B\cap C)$ thrice. So to include $n(A\cap B\cap C)$ in the union, we need to add at last. Also, $n(A) + n(B) + n(C)$ covers $n(A\cap B) + n(B\cap C) + n(C\cap A)$ twice. So we subtract it once and make the formula.

Let us see one more diagram:



Here, A is a subset of B. So A lies inside the set B. In this case,

clearly from the diagram $A \cap B = A$ and $A \cup B = B$.
Hence $n(A \cap B) = n(A)$ and $n(A \cup B) = n(B)$.
Also $n(B-A) = n(B) - n(A \cap B) = n(B) - n(A)$ and
$n(A-B) = n(A) - n(A \cap B) = n(A) - n(A) = 0$.

Examples:

1. If $n(U) = 500$, $n(A) = 150$, $n(B) = 300$, $n(A \cup B) = 380$, find $n(A \cap B)$, $n(A-B)$, $n(B-A)$, $n(A \cup B)'$.

Here,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Or $380 = 150 + 300 - n(A \cap B)$

Or $n(A \cap B) = 150 + 300 - 380$

$=70$.

Again,

$n(A- B) = n(A) - n(A \cap B)$

Or $n(A- B) = 150 - 70$

$= 80$

Also,

$n(B - A) = n(B) - n(B \cap A)$

Or $n(B - A) = 300 - 70$

$= 230$.

Now,

$n(A \cup B)' = n(U) - n(A \cup B)$

Or $n(A \cup B)' = 500 - 380$

$= 120$.

2. If $n(U) = 500$, $n(A) = 150$, $n(B) = 300$ and $A \subset B$
find $n(A \cap B)$, $n(A-B)$, $n(B-A)$, $n(A \cup B)$, $n(A \cup B)'$.

Here,

If $A \subset B$ then $A \cap B = A$.

So, $n(A \cap B) = n(A)$

$\therefore n(A \cap B) = 150$

Again,

$n(A - B) = n(A) - n(A \cap B)$

Or $n(A - B) = n(A) - n(A) = 0$

Also

$n(B - A) = n(B) - n(B \cap A)$

Or $n(B - A) = n(B) - n(A)$

$= 300 - 150$

$= 150$.

Now

If $A \subset B$ then $A \cup B = B$.

So, $n(A \cup B) = n(B)$

$\therefore$ $n(A \cup B) = 300$.

And

$n(A \cup B)' = n(U) - n(A \cup B)$

Or $n(A \cup B)' = n(U) - n(B)$

$\therefore$ $n(A \cup B)' = 500 - 300$

$= 200$.

Class activities and class works:

1. If $n(U) = 100$, $n(A) = 75$, $n(B) = 40$, $n(A \cup B) = 80$, find $n(A \cap B)$, $n(A - B)$, $n(B - A)$ and $n(A \cup B)'$.

2. If $n(A) = 37$, $n(B) = 50$ and $A \subset B$, find $n(A \cup B)$, $n(A \cap B)$, $n(A - B)$ and $n(B - A)$.

3. In a certain village in Nepal, all people speak Nepali or both. If 60% speak Nepali and 50% speak Newari, how many speak both languages?

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