Second week lessons.

(Divided into 3 lectures of 50 minutes each)

Lecture – 4(50 minutes)

Assignment on set theory with some hints

1. If U = {a, b, c, d, x, y, z}, A = {a, b, c, d}, B = a, x, y, z}, C = {a, c, x, y} find the following

   i) $A\cup B$ ii) $B\cap C$ iii) A^c iv) B - C v) $A'\cap B'$ vi) $B'\cup C'$

   [HInts: use the concept of union, intersection, difference and complements]

2. Given U = {1, 2, 3......., 15}, A = {x: x ≥ 8}, B = {x: x ≤ 4}, C = {x: 4 < x < 12}. Find the following

   i) $A\cup B$ ii) $(A\cup C)'$ iii) A - C iv) B - C v) $A'\cap B'$ vi) $B'\cup C'$

   [HInts: x≥8 means the numbers more than or equal to 8. It collects the numbers 8, 9, ...]

3. If n(U)=100, n(A) = 75, n(B) = 40, $n(A\cup B)$ =80, find $n(A\cap B)$, n(A- B), n(B - A) and $n(A\cup B)^c$.

4. If n(U)=100, n(A) = 75, n(B) = 40, and $B\subset A$, find $n(A\cup B)$, $n(A\cap B)$, n(A- B), n(B - A) and $n(A\cup B)^c$.

5. If A = { 1, 2, 3, 4} and B = { 2, 3, 5, 7}, find $A\Delta B$. [Hint: $A\Delta B = (A-B)\cup(B-A)$]

6. If U = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and B ={b:b is a positive even integer less than 10}, define B'.

7. If U equals the set of students in a mathematics class and P is the students who fail the course, define P'.

8. Given U = {2, 4, 6, 8, 10, 12, 14, 16, 18}, A = 4, 8, 16}, B = 2, 4, 6, 8, 10} draw a Venn diagram representing it.

9. If U = {x: x is a negative integer greater than -11}, A = {a| a is a negative odd integer greater than -10}, and

   B = {b: b is a negative integer greater than -6}, draw a Venn diagram representing the sets.

10. If U = {a, b, c, d, e, f, g}, A = {a, b, c, d, e}, B = {a, c, e, g} and C = {b, e, f, g}; verify that

    i) $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

    ii) $A - (B\cap C) = (A - B)\cup(A - C)$

    iii) $(A\cup B)' = A'\cap B'$

    [Hints: find the collection of elements in left hand side expression and also in right hand side expression. Then check whether they are exactly same or not]

11. In a BBA 1st terminal examination, 52% failed in Business Mathematics, 42% in Account, 28% in Sociology, and 35% in Business Mathematics and Account, 20% in Account and Sociology, 18% in Business Mathematics and Sociology and 14% in the three subjects. Then find

a) What percent passed in all three subjects. b) What percent failed in exactly two.

b) What percent failed in exactly one subject.

[Hints: all three means the intersection of all sets, exactly two mean sum of only two parts and exactly one means the sum of exactly one parts]

12. In a survey of 36 students, 18 like Economics, 19 Mathematics, 16 Account. If 8 like Economics and Mathematics, 5 like Mathematics and Account and 7 like Account and Economics, find the number of students who like all three subjects. [Hints: Since there is no one who dislike the three subjects, the union of three sets is 36]

13. In a group of students, 18 read Economics, 19 read Mathematics, 16 read Accountancy, 6 read Economics only, 9 read Mathematics only, 5 read Economics and Mathematics only and 2 read Mathematics and Accountancy only. Find:

    i) How many read all three subjects? ii) How many read Economics and Accountancy only?

    iii) How many read Accountancy only? iv) How many students are there all together?

14. A group of 1000 people was asked if they had purchased three different newspapers: Kantipur, Gorkhapatra and Rajdhani. The following data were gathered.

    220 people had purchased Kantipur. 175 had purchased Gorkhapatra. 150 had purchased Rajhani. 50 had purchased both Kantipur and Gorkhapatra. 75 had purchased both Kantipur and Rajdhani. 60 had purchased both Gorkhapatra and Rajdhani. 20 has purchased all three. Find how many had purchased

    i) Kantipur only? ii) Gorkhapatra and Rajdhani only? iii) at least one news paper? iv) none of the three?

15. In a town of 50,000 population in Nepal, 28,000 read Gorkhapatra, 5,000 read Kantipur and 1,000 read both. What percentage read neither Gorkahapatra nor Kantipur?

16. Of 100 students in and examination, 42 offered Mathematics, 35 offered Physics and 30 offered Chemistry. 20 offered none of these subjects, 9 offered Mathematics and Chemistry, 10 offered Physics and Chemistry and 11 offered Mathematics and Physics. Find the number of students

    i) offering all three subjects. ii) Mathematics only iii) Physics and Chemistry only.

17. A major TV network surveyed 50,000 viewers to determine their viewing habits during the past month. They found that 29,000 viewers had watched a sports events. 25,000 viewers had watched a news show. 28,000 viewers had watched a TV movie. 16,000 viewers had watched both sports and news shows. 15,000 viewers had watched both news shows and TV movie. 18,000 viewers had watched both sport shows and TV movie.

10,000 viewers had watched all three types of shows. Determine the number of viewers who had watched

i) news shows only. ii) Sports shows only. iii) none of the three types of programs.

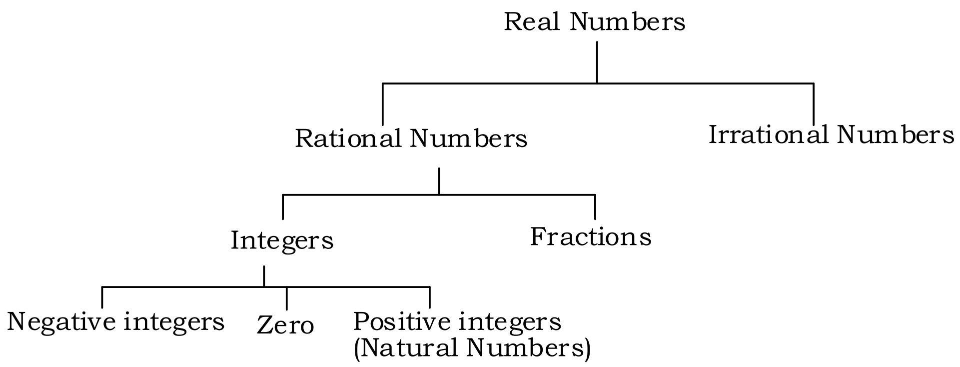
Lecture 5 (50 minutes)

Real numbers:

a) Discussion on Natural numbers, Integers, Rational numbers, Irrational numbers and real numbers making a tree diagram.
b) Real numbers and number lines.
c) Class works

A. Discussion on Natural numbers, Integers, Rational numbers, Irrational numbers and real numbers making a tree diagram.

The set of all rational and irrational numbers taken together form a new system of numbers known as real number system. The diagram for the real number system is given below.



Note:

1. ℤ = The set of integers.

   Example: ...... -3, -2, -1, 0, 1, 2, 3,

2. $\mathbb{Z}^+$ The set of positive integers. It is also called the set of natural numbers numbers, denoted by the letter ℕ.

   Example: ... ... 1, 2, 3, —

3. $\mathbb{Z}^-$ The set of negative integers.

   Example: ... ... -1, -2, -3 ....

4. ℚ The set of rational numbers, which are in the form p/q with p and q integers and q≠0.

   Example: 0/1, 1/1, -1/1, 2/3, 0.45,.03333..., -3/5 and so on.

5. A real number not expressible as an integer or quotient of integers is an irrational number. In other words a non rational number is an irrational number.

   Example: √2, 1.732..., π etc.

[Note that 1.732 is rational but 1.732... is irrational because terminating decimal numbers are all rational numbers but non-terminating and non repeating decimal numbers are irrationals. Although 1.323232... is non-terminating, it is repeating with the pattern of 32. So 1.323232... is rational. We can change this into p/q form by using a rule of geometric series]

√2 is an irrational number.

Proof:

If possible , suppose √2 is a rational number such that √2 = p/q, where p and q are integers having no common factor and q ≠ 0. ... ... (1)

Then √2 = p/q

$\Rightarrow 2q^2 = p^2$ ... ... [i]

$\Rightarrow P^2$ is an even and hence p is even. ... ... (2)

Suppose, p = 2r, where r is an integer.

Then from (i) we get

$2q^2 = (2r)^2$,

$\Rightarrow q^2 = 2r^2$.

$\Rightarrow q^2$ is an even and hence q is even ... . (3)

Therefore, from (2) and (3) we get that p and q both are even and hence have at least one common factor 2. This contradicts (1). So, our supposition was wrong.

That is, if we suppose √2 a rational number, we will get contradictory result.

Hence √2 cannot be supposed as a rational number.

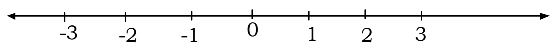
So, √2 is an irrational number.

B. Real numbers and number lines.

Real number line is a straight horizontal line on which points are identified with real numbers, points identified with successive integers usually being spaced at unit distance apart.

Example:

The real number line is as follows:



[Note: there are infinitely many real numbers between any two real numbers. The points in left are less than the points in right]

C. Class work

1. Write the notations for the set of natural numbers, integers, rational numbers and real numbers.
2. Prove √5 is irrational number.
3. Give some examples of rational and irrational numbers.
4. Give two examples of each of real, rational, irrational, integers and natural numbers.

Lecture 6 (50 minutes)

a. Inequalities
b. Intervals and their union, intersection, difference.
c. Class works

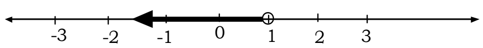
A. Inequality.

Suppose a and b are two real numbers. a is said to be less than b, if a - b is negative. In symbols, we write $a<b$. When a - b is negative, we may write this as $b>a$. Here $<$ stands for "less than" and $>$ stands for "greater than". If b - a is zero, a and b are equal. We write this as a = b. If a is less than or equal to b, we write $a \leq b$ and if a is greater than or equal to b, we write $a \geq b$. We summarize these symbols below:

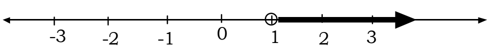
Symbol	Meaning
$<$	Less than
$>$	Greater than
$\leq$	Less than or equal to
$\geq$	Greater than or equal to

Representation of the inequalities in real number line for different cases are given below.-

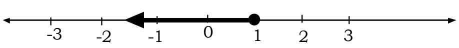
1. For the type $x < a$. *Example: $x < 1$*



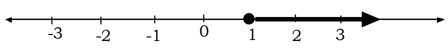
2. For the type $x > a$. *Example:* $x > 1$



3. For the type $x \leq a$. *Example:* $x \leq 1$



4. For the type $x \geq a$. *Example:* $x \geq 1$



B. Intervals and their union, intersection, difference.

Given any two points a and b (where a < b) on the real line, the set of all points between a and b is known as an interval, and the points a and b are called the end points. An interval may or may not contain the end points. Depending upon how the end points occur in an interval, we may have four different types of intervals. They are:

Closed	$[a, b] = \{x: a \le x \le b\}$
Open	$(a, b) = \{x: a < x < b\}$
Left closed	$[a, b) = \{x: a \le x < b\}$
Left open	$(a, b] = \{x: a < x \le b\}$

The set [a, b] containing all real numbers between a and b including both the end-points is called a closed interval; the set (a, b) without the end-points is called an open interval; and the remaining sets are called half open or half closed intervals.

We summarize the above ideas below:

Set	Notation	Graph	Name
$\{x: a \le x \le b\}$	[a, b]		Closed interval
$\{x: a < x < b\}$	(a, b)		Open interval
$\{x: a \le x < b\}$	[a, b)		Right open interval
$\{x: a < x \le b\}$	(a, b]		Left open interval

[Note: in number line, the included end point is represented by a filled circle while the non included ends are represented by just a circle without blackening]

Since intervals are the sets of real numbers, we can find their union, intersection and difference. It will be clear by the following examples:

Example

If A = [-3,2) and B = [-2, 3], compute (a) $A \cup B$ (b) $A \cap B$ (c) A - B

Here,

(a) $A \cup B = [-3, 2) \cup [-2, 3]$

$= \{x: -3 \le x < 2\} \cup \{x: -2 \le x \le 3\}$

$= \{x: -3 \le x \le 3\}$

$= [-3, 3]$.

[Here, the first set A contains all real numbers from -3 to 2 and second set B contains the real numbers from -2 to 3. so the real numbers from -2 to 2 are in intersection. Only A contains the real numbers in [-3, -2), only B contains the real numbers in [2, 3] and intersection contains the real numbers in [-2, 2). we can show them in a real number line. Then the results of the operations will be clear.]

(b) $A \cap B = [-3, 2) \cap [-2, 3]$

$= \{x: -3 \le x < 2\} \cap \{x: -2 \le x \le 3\}$

$= \{x: -2 \le x < 2\}$

$= [-2, 2)$.

(c) $A - B = [-3, 2) - [-2, 3]$

$= \{x: -3 \le x < 2\} - \{x: -2 \le x \le 3\}$

$= \{x: -3 \le x < -2\}$

$= [-3, -2)$.

C. Class works.

1. Represent the following intervals in the set builder form:
   a) $B=[1,3)$ b) $S = [1, 3]$ c) $R = (1, 3)$
2. Let $A = [-1, 3)$ and $B = [2, 4]$, compute:
   a) $A \cup B$ b) $A \cap B$ c) $A - B$
3. Let $A = (-2, 3]$ and $B = (2, 4]$, compute:
   a) $A \cup B$ b) $A \cap B$ c) $A – B$
4. Represent the following inequalities in number lines.
   i) $x < 4$ ii) $x \le -4$ iii) $x > 5$ iv) $x \ge -5$