Fifth week lessons.

(Divided into 3 lectures of 50 minutes each)

Lecture – 13 (50 minutes)

a) Class works on Permutation and combination.
b) Assignment

A. Class works

Permutation

1. In how many ways can 5 prizes be distributed among 4 students, when each student may receive any number of prizes ?

   [The one prize can be given to any one of the 4 students. So For that prize 4 possible distributions are there. Similarly, the another prize also has 4 possible distributions. So, total number of distributions= 4.4.4.4.4 = 4^5 = 1024]

2. A boy has 6 pockets. In how many ways can he put 5 marbles in his pocket ?

   [Find out the reason why the answer is 6^5?

3. In how many ways can 11 members of a committee sit at a round table?

   If the secretary and the joint secretary are always the neighbor of the president, find the number of arrangements.

4. In how many different ways can a garland of 16 different flowers be made?

Combination

20. In how many ways a student can choose 5 courses out of 9 courses, if 2 courses are compulsory for every student.?
21. If there are 12 persons in a party and if each two of them shakes hands with each other, how many handshakes happen in the party ? [$^{12}C_2$ why?]
22. A question paper has two parts, part A and part B, each containing 10 questions. If a student has to choose 8 from part A and 5 from part B, in how many ways can he choose the questions?
23. Out of 6 teachers and 8 students, a committee of 11 is to be formed. In how many ways can this be done, if the committee contains :

    i) exactly 4 teachers,

    ii) at least 4 teachers.

24. A man has 6 friends. In how many ways can he invite one or more to a party?

B. Assignment

1. A committee is to be chosen from 12 men and 8 women and is to consist of 3 men and 2 women. How many committees can be formed?

2. In how many ways can four boys and three girls be seated in a row containing seven seats
   (a) if they may sit anywhere

   (b) if the boys and girls must alternate?

3. How many numbers between 4000 and 5000 can be formed with the digits 2, 3, 4, 5, 6? How many of these numbers are divisible by 5?

4. How many 6-digit telephone numbers can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 if each number starts with 35 and no digit appears more than once?

5. In how many ways can the letters of the word MONDAY be arranged? How many of these arrangements do not begin with M? How many of these begin with M and do not end with Y? [Answers 720, 600, 96]

6. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady?

7. How many ways can the letters of the word ARRANGE be arranged, so that R's do not come together?[900]

8. A candidate is required to answer 6 out of 10 questions which are divided into 2 groups each containing 5 questions and he is not permitted to attempt more that 4 from any group. In how many different ways can he make up his choice?

9. A person has 14 acquaintances of whom 10 are relatives. In how many ways can he invite 8 guests so that 5 of them may be relatives?

10. There are 5 boys and 3 girls. In how many ways can they stand in a row so that

    (i) they may stand anywhere.

    (ii) no two girls are together.

11. In how many ways can a person wear 3 pants (different colours), 4 shirts(different in colours) and 2 ties(different in colours)?

12. A political candidate wishes to visit eight different states. In how many different orders can she visit these states?

13. A portfolio management expert is considering 30 stocks for investment. Only 15 stocks will be selected for inclusion in a portfolio. How many different combinations of stocks can be considered?

14. A major research foundation is considering funding a set of medical research projects. Fifteen applications have been submitted, but only six will receive funding. How many different sets of projects could be funded?

15. If $^{18}C_r = {}^{18}C_{r+2}$, find r and $^rC_3$. **Note:** If $^nC_p = {}^nC_q$, then p + q = n

16. A committee of 7 members is to be chosen from 6 Chartered Accountants, 4 Economists and 5 Cost Accountants. In how many ways can this be done if in the committee, there must be at least one member from each group and at least 3 Chartered Accountants.

Lecture – 14 (50 minutes)

a) Linear equations in one variable.
b) Linear equations in two variables.
c) Graphical representation of linear equations.
d) Class works

A. Linear equations in one variable

If an equation contains one variable and if the variable occurs to the first degree, the equation is called a linear equation in one variable. It is of the form ax + b = 0, where a and b are real numbers and a ≠ 0. A linear equation is also called a first degree equation.
For example, 3x + 6 = 15 is a linear equation in variable x.

Example:
The sum of three consecutive natural numbers is 13 greater than twice the smallest, find the numbers.
Solution:

Let the first number be x.
Then the second is x +1 and the third is x+1+1 or x + 2.
Now
According to the question,
$x + (x+1) + (x+2) = 13 + 2x.$
or $3x + 3 = 13 + 2x$
or $3x - 2x = 13 – 3$
or $x = 10$.

Hence the numbers are 10, 11 and 12.

B. Linear equations in two variables

A linear equation involving two variables x and y has the standard form ax + by = c, where a, b, c are real numbers and a, b cannot both equal to zero. For example 2x + 4y = 7 is a linear equation of two variables.

Example:
Given the equation 2x + 4y = 8

a) Determine any pair of values that satisfies the given equation.
b) Determine the pair of values which satisfies the given equation for x = -2.
c) Determine the pair of values of which satisfies the given equation for y = 0.

Solution

a) For any pair of values of x and y we can suppose any number as a value of x and find the corresponding value of y. Suppose x = 1, then the equation becomes

$$2.1 + 4y = 8$$
or $4y = 8 - 2$
or $4y = 6$
or $y = 6/4$
$= 1.5$

Thus a pair of values satisfying the given equation is x = 1 and y = 1.5. i.e. (1, 1.5)

b) when x = -2,

$$2.(-2) + 4y = 8$$
or $4y = 8 + 4$
or $4y = 12$
or $y = 12/4$
$= 3$

Thus pair of values satisfying the given equation is x = -2 and y = 3. i.e. (1, 1.5)

c) when y = 0,

$$2x + 4.0 = 8$$
or $2x = 8$
or $x = 8/2$
or $x = 4$

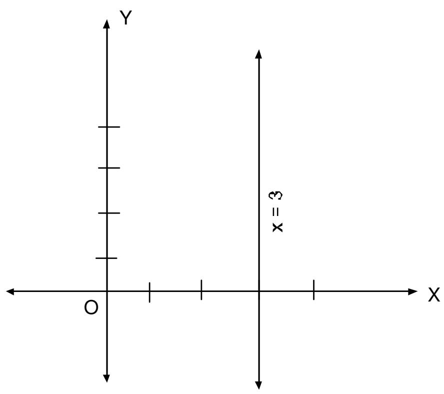
Thus pair of values satisfying the given equation is x = 4 and y = 0. i.e. (4, 0)

[Note: In the above example, the number of variables is two and the number of equations is one. In such case, there exist infinitely many solutions]

C. Graphical representation of linear equations.

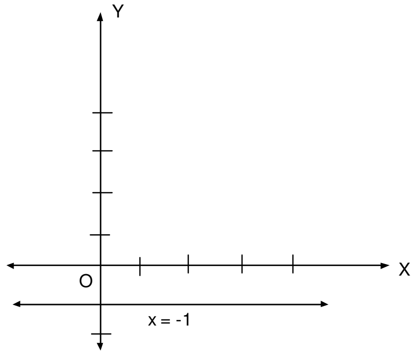
The linear equations can be represented graphically. Their graphs are straight lines. If the equation contains only one variable x, then its graph is a straight line parallel to y axis. If the equation contains only one variable y, then its graph is a straight line parallel to x axis. To draw the linear equation of two variables, we find at least two points satisfying the equation as in above example. Then we plot the points and join them and extend in both sides.

Example1:
Draw the graphs of 2x – 6 = 0
Solution:
2x – 6 = 0 gives
x = 3.
So we draw the graph of x = 3. Its graph is a straight line which is parallel to y axis and 3 units far from origin as shown in the diagram alongside.



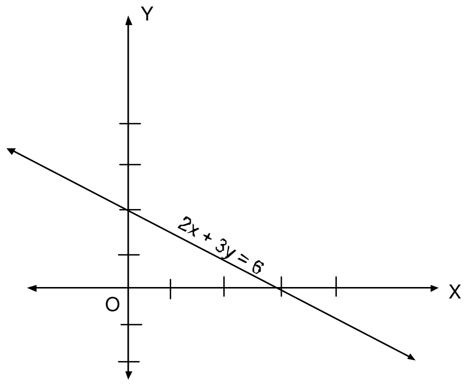
Example2:
Draw the graphs of 2y + 2 = 0
Solution:

2y + 2 = 0 gives
y = -1.
So we draw the graph of y = -1.
Its graph is a straight line which is parallel to x axis and -1 unit far from origin as shown in the diagram alongside.



Example2:
Draw the graphs of 2x + 3y = 6
Solution:

First we find any two points satisfying the given equation. when x = 0, by calculation we get y = 2. So (0, 2) is a point. And when y = 0, we get x = 3. So (3,0) is another point. The graph is a straight line which passes through the points calculated above. Its graph is shown in the diagram alongside.



D. Class works

1. Solve the following equations
   i) 3x+5 = 11 ii) 4x – 7 = 5 iii) -2x + 3 = -2 iv) 5y - 11 = -8
2. Find three pairs of points which satisfy the following equations
   i) 3x + 2y = 6 ii) x + 3y = 7
3. Draw the graphs of the following linear equations.
   i) 3x-5 = -11 ii) 2y = 7 iii) x + 3y = 7

Lecture – 15 (50 minutes)

a) Elimination method to solve the system of linear equations.
b) Graphical method to solve the system of linear equation of two variables.
c) Class works

A. Elimination method to solve the system of linear equations.

In elimination technique we apply algebraic operations to eliminate one or more variables from the given system in order to get the solution of the system.
The eliminating procedure can be genrralized as follows for two equations containing two variables.

i) Multiply (if necessary) the equations by constants so that the coefficients on one of the variables are the negative of one another in the two equations.
ii) Add the two resulting equations.

iii) Then one variable will be cancelled and an equation containing other single variable will be obtained. From that equation, the value of one variable will be obtained. Putting the value of that variable in one of the given two equations, the value of other variable is calculated.

iv) Note: If the result after adding two equations is an identity like 0 = 0, 4= 4, then the given equations have infinitely many solutions.
If the result is false like 5 = 0, the equations are inconsistent and there is no solution.

Example:

Solve by elimination method 2x + 5y = 11 and 3x+4y = 13.

Solution:

To make the coefficients of x in both equations same and opposite in sign, we multiply first equation by three and the second equation by -2 and the equations become

$$6x + 15y = 33$$
$$-6x - 8y = -26$$

Adding $0 + 7y = 7$

So $y = 1$.

Putting the value of y in first equation (2x + 5y = 11) we get

$2x + 5.1 = 11$

or $2x = 11 - 5$

or $2x = 6$

so $x = 3$.

Hence the values of x and y are 3 and 1 respectively.

B. Graphical method to solve the system of linear equations.

From above we know how the draw the linear equations in a graph paper. If we draw two linear equations containing two variables in a graph, they intersect at a point (if they are not parallel). The point of intersection is the solution of the equations. i.e. we write the coordinates of the point of intersection of two lines which are the values of the variables satisfying both the given equations.

Example

Solve by elimination method 2x + 5y = 11 and 3x+4y = 13.

Solution:

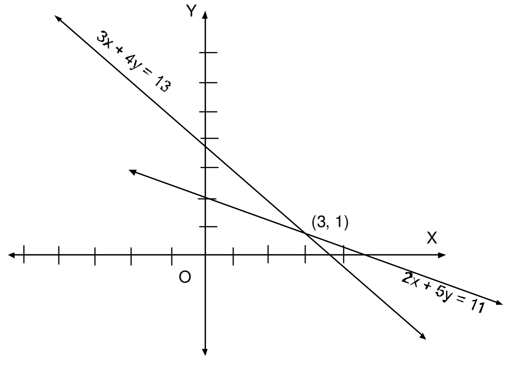
For the first equation 2x + 5y = 11, when x = -2, y = 3 and when y = 0, x = 5.5

So, the line passes through (-2, 3) and (5.5, 0).

Similarly, for the second equation 3x + 4y = 13, when x = -1, y = 4 and when x = -5, y = 7

So, the second line passes through (-1, 4) and (-5, 7).

Now the graph of the equations is as follows



In the graph, the lines intersect at (3, 1). So, the solution of two equations is $x = 3$ and $y = 1$.

C. Class works

1. Solve the following equations by elimination method.

   i) $2x + 3y = 7,\ x - y = 1$ ii) $-x + 2y = 1,\ 3y + 2x = 12$

2. Solve the following equations by graphical method.

   i) $2x + 3y = 7,\ x - y = 1$ ii) $-x + 2y = 1,\ 3y + 2x = 12$