Sixth week lessons.

(Divided into 3 lectures of 50 minutes each)

Lecture – 16 (50 minutes)

a) Assignment

A. Assignment

1. Solve the following equations:

   i) $3x + 4y = 12$ and $4x + 3y = 12$ ii) $\frac{5}{x-4} = \frac{6}{x-3}$ iii) $-2x + 5 = 9$

2. The sum of three consecutive positive numbers is 18. Find the numbers.

3. Solve the following system of lines by elimination method
   $x + y = 3$, $2x - y = 12$

4. Solve the following equations by elimination method

   $2x - 5y = -2$, and $x + y = 6$.

5. Suppose a toy manufacturer has fixed costs of Rs 5,000. In addition, there are variable cost of Rs 5 per toy.

   i) Find the total cost function for x toys.

   ii) How many toys may be produced at a cost of Rs 10,000?

   iii) Suppose the selling price of a toy is Rs 10, find the profit function.

   iv) Find the profit generated by 8000 toys.

   [Hints: Cost function = fixed cost + variable cost, Profit = Selling price – cost]

6. Solve by graphical method

   $x + y = 3$, $2x - y = 12$

7. Solve by graphical method

   $2x - 5y = -2$, and $x + y = 6$.

8. The value of a truck is estimated by the function $V = 20{,}000 - 3{,}000t$, where V equals the value stated in dollars and t equals the age of the truck expressed in years.

   a) What is the value after 3 years? b) When will the value equal 0?

Lecture – 17 (50 minutes)

a) Definition and examples of function.
b) Domain and Range of the functions of the types f(x) = ax^2+bx+c and f(x) = $\dfrac{1}{x-a}$
c) Class works

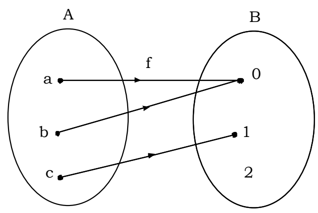
A. Definition and examples of function

Definition of function:

A function from a set A to a set B is a relation (or rule), which associates each element of A with unique element of B.
Symbolically, we write
f : A→ B, to mean "f is a function from A to B".

> **Note**- If f : A→ B is a function from A to B and x in A has corresponding image y in B, then we write f(x) = y. In this case y is called the **image** of x and x is called the **pre-image** of y. The set A is called the **domain** and the set B is called the **co-domain.** The collection of elements of B which have pre image in A forms a set and this set is known as **range** of the function f So, the collection of all f(x) is the **range** of f.

The arrow diagram given below represents a function f defined from A to B.
In this diagram, the domain = {a, b, c}
co-domain = {0, 1, 2}
range = {0, 1}



This is a function because the function satisfies two basic conditions to be a function.
They are

i) Every element of A has an image. (i.e. every element of domain is associated with the elements of co-domain)
ii) Image of every element is unique. (i.e. every element does not have more than one image)

B. Domain and Range of the functions of the types $f(x) = ax^2+bx+c$ and $f(x) = \frac{1}{x-a}$

As mentioned above, the **domain** of a function f:A→B is the collection of the elements of the set A and the **range** of the function is the collection of the images of f.
If f : **R** →**R** is defined as = f(x) then the collection of all possible values of x forms the domain and the collection of all corresponding values of y forms range of the function.
For example If y = f(x) = √x, then the values of x cannot be negative. So the domain contains the real numbers starting from zero to infinity. Since the given root is always positive, the range also contains the real numbers starting from zero to infinity. Hence the domain is [0, ∞) and range is [0, ∞).

Some important facts to find domain and range of a function defined from the set of real numbers to itself are summarized in the following note.

Note:

1. If the given function contains no root and no denominator, the domain may contain all the real numbers.
2. For $\sqrt{\text{an expression}}$, the expression ≥ 0.
3. For $\frac{\text{something}}{\sqrt{\text{an expression}}}$, the expression > 0.
4. For $\frac{\text{something}}{\text{an expression}}$, the expression ≠ 0.
5. If y = $\sqrt{\text{an expression}}$, the values of y (the elements of range) cannot be negative because the given square root gives always positive results.

To find domain or range, we use the above mentioned facts. To find domain we use the conditions directly in the given function and find the collection of possible values of x and to find range, we suppose y = f(x) and simplify to get x = f(y) then use the same (above mentioned) facts to find the possible values of y. From fact 5, we have to know that if the question contains positive square root then range contains only positive numbers.

For the range of a quadratic function, it is easier to calculate by completing the square.
Example 1:
Find the domain and range of $f(x) = x^2 + 3x + 6$
Solution:

Since there is no square root and no denominator, the x can be any real number.
So, domain is (-∞, ∞).
For range

$f(x) = x^2 + 3x + 6$
$= x^2 + 2.x.{}^3/_2 + ({}^3/_2)^2 + 6 – ({}^3/_2)^2.$
$= (x +{}^3/_2)^2 + 6 – {}^9/_4.$
$=(x +{}^3/_2)^2 + {}^{15}/_4.$

this is always more than or equal to ${}^{15}/_4$ because the whole square is always positive whose minimum value is 0. So $f(x) \geq {}^{15}/_4$.
Hence range = $[{}^{15}/_4, \infty)$

Example 2:

Find the domain and range of f(x) = $\frac{1}{x-4}$

Solution:

For domain

Since there is denominator, the value of denominator cannot be zero otherwise the functional value will be undefined. so we take

$x - 4 \neq 0$

Or $x \neq 4$.

So domain can take all real numbers except 4.

hence the domain = $\mathbb{R} - \{4\}$ or $(-\infty, 4)\cup(4, \infty)$.

For range,

let $y = \frac{1}{x-4}$

or $x - 4 = \frac{1}{y}$

or $x = \frac{1}{y} + 4$

or $x = \frac{1+4y}{y}$, here the denominator cannot be zero so $y \neq 0$.

hence the range = $\mathbb{R} - \{0\}$ or $(-\infty, 0)\cup(0, \infty)$.

Example 3:

Find the domain and range of $y = \frac{2x+1}{x-4}$

Solution:

For domain

The expression $\frac{2x+1}{x-4}$ is well defined if

$x - 4 \neq 0$

Or $x \neq 4$

Hence the domain is $\mathbb{R} - \{4\}$ or $(-\infty, 4) \cup (4, \infty)$

For range

$y = \frac{2x+1}{x-4}$

Or $y (x - 4) = 2x + 1$

Or $xy - 4y = 2x + 1$

Or $xy - 2x = 1 + 4y$

Or $x(y - 2) = 1 + 4y$

Or $x = \frac{1+4y}{y-2}$, which is well defined if

$y - 2 \neq 0$

Or $y \neq 2$

Hence the range is $\mathbb{R}$ - {2} or $(-\infty, 2) \cup (2, \infty)$

C. Class works

Find the domain and range for the following functions:

i) $f(x) = x^2 - 4x + 7$.   ii) $f(x) = x^2 - 10x + 6$.   iii) $f(x) = 3x^2 - 4x + 6$.

iv) $f(x) = \frac{2}{x+3}$ v) $f(x) = \frac{3x-1}{x-2}$

Lecture – 18 (50 minutes)

a) Domain and range of the functions of the type $f(x) = \sqrt{x-a}$ and $f(x) = \frac{1}{\sqrt{x-a}}$

b) Class works

A. Domain and range of the functions of the tye f(x) = $\sqrt{x-a}$ and f(x) = $\frac{1}{\sqrt{x-a}}$

As mentioned in the above note, the expression under the square root must be positive. Using that rule, we find domain. The range of the square root function cannot contain negative numbers. So we take positive numbers only. Let us see some examples.

Example 1:

Find the domain and range of $f(x) = \sqrt{x-2}$

Solution

For domain:

Since there is square root, we must have $x - 2 \geq 0$.

or $x \geq 2$

So, domain = $[2, \infty)$

For range, let y = f(x)

or $y = \sqrt{x-2}$

or $y^2 = x - 2$

or $x = y^2 + 2$.

here y can be any real number. But the given function is positive square root function, so range cannot take negative part. hence omitting the negative part from the set of real numbers, the required range = $[0, \infty)$.

Example 2:

Find the domain and range for $f(x) = \frac{1}{\sqrt{x+3}}$

Solution:

Since there is square root in denominator, we must have

$x + 3 > 0$

or $x > -3$

hence domain = $(-3, \infty)$

For range, let $y = f(x)$

or $y = \frac{1}{\sqrt{x+3}}$

or $y^2 = \frac{1}{x+3}$

or $x + 3 = \frac{1}{y^2}$

or $x = \frac{1}{y^2} - 3$

or $x = \frac{1-3y^2}{y^2}$. here, $y^2 \neq 0$, or $y \neq 0$.

But the given function contains positive square root. So range cannot take negative numbers. Hence the range is $(0, \infty)$.

B. Class works

Find the domain and range of the following functions:

i) $y = \sqrt{x-1}$ ii) $f(x) = \sqrt{1-x}$ iii) $y = \frac{1}{\sqrt{x-1}}$ iv) $f(x) = \frac{1}{\sqrt{1-x}}$ v) $f(x) = \frac{x+1}{x-2}$