Eighth week lessons

Function (continued) & Quadratic Equations

(Divided into 3 lectures of 50 minutes each)

Lecture – 22 (50 minutes)

a) Break even analysis
b) Supply, Demand and market equilibrium.
c) Class works

A. Break even analysis

In the break even analysis, we determine the level of output that would result in a zero profit, i.e. no profit no loss. This level of output is called the break even point. That is, the break even point is the point at which the revenue is equal to the cost.

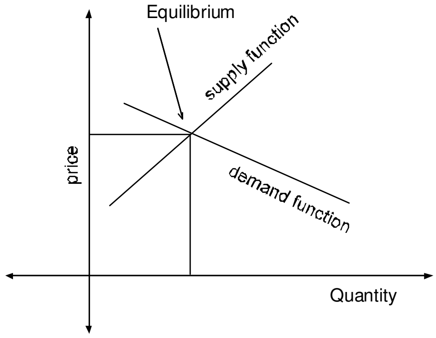
Example:
a firm sells a product for Rs 45 per unit. Variable costs per unit are Rs 33 and fixed cost is Rs 450,000. How many units must be produced and sold in order to have break even?
Solution:
Suppose the number of units produced and sold is x. Then the total cost C is given by
$C = 450,000\ 33x$
Revenue (R) = 45x
For break even, we take R = C
or $45x = 450000+33x$
or $x = 37500$
Hence the firm should produce and sell 37500 units for break even.

B. Demand, Supply and Market Equilibrium

Demand is the quantity of a product or service desired by buyers. That is, the quantity demanded is the amount of a product people are willing to buy at a certain price

Supply represents how much the market can offer. That is, the quantity supplied is the amount of the product which is supplied in the market to get certain price.

When the demand and supply are equal, the economy is said to be at equilibrium. In the mathematical graph, the intersection of demand and supply functions is the point of equilibrium.



Example:
Find the market equilibrium price for the demand: 2p = -q + 56 and supply: 3p – q = 34.
Solution:
the demand function is 2p = -q + 56

$$\text{or } p = \frac{-q+56}{2}$$

the supply function is 3p – q = 34.

$$\text{or } p = \frac{q+34}{3}$$

At equilibrium, the demand price is equal to supply price.

$$\text{so } \frac{-q+56}{2} = \frac{q+34}{3}$$

or -3q + 168 = 2q + 68
or 5q = 100
or q = 20 and corresponding price = is 18.

C. Class work

1. A manufacturer sells belts for Rs 12 per unit. The fixed costs are Rs 1600 per month, and the variable costs are Rs 8 per unit.
   a) Write the equations of the revenue and cost functions.
   b) Find the break even point.
   c) Write profit function.
   d) Set profit equal to zero to solve x. Compare this x-value to the answer of part b.
2. If the demand for a pair of shoes is given by 2p + 5q = 200 and the supply function for it is p – 2q = 10, compare the quantity demanded and the quantity supplied when the price is Rs 60.

Lecture – 23 (50 minutes)

a) Exercise of the problems related to functions.
b) Assignment

A. Exercise of the problems related to functions.

1. Determine the domain and range of the following functions.
   i) f(x) = 2x + 5 ii) $f(x) = \sqrt{x-4}$ iii) $f(x) = \frac{1}{\sqrt{3-x}}$
2. If f(x) = 7x – 2 and $g(x) = x^2$, find the following composite functions.
   i) $f_o g(x)$ ii) $g_o f(x)$ iii) $f^{-1}{}_o g(x)$ iv) $g_o f^{-1}(x)$
3. If the demand function and supply function for Z-brand phones are p+2q = 100 and 35p – 20q = 350 respectively, find the equilibrium price and corresponding demand.

B. Assignment

1. What are injective, surjective and bijective functions? Describe with examples.

2. If $f(x) = 2x^2 + 3$, find f(2) and f(x + 3)).

3. Let A = {-1, 0, 2, 4, 6} and f: A→**R** be defined by

   $y = f(x) = \frac{x}{x+2}$, find the range of f.

4. Determine whether each of the following functions is injective or surjective.

   a) Let A be the set of odd positive integers.

   A function f: **A→B** is defined by f(x) = 2x + 1.

   b) Let A = {a, e, i, o, u} and B = {d, m, r} and a function f: A→B is defined by f(a) = f(e) = d, f(i) = m and f(o) = f(u) = r.

5. Let f: **R→R** be defined by f(x) = 2x - 3, x∈**R**, Find a formula that defines the inverse function f^{-1}.

6. Let f: **R→R** and g: **R→R** be defined by f(x) = x + 1 and $g(x) = x^3$. Find $(g_o f)(x)$ and (fg)(x).

7. Let f: **R→R** and g: **R→R** be defined by f(x) = 2x + 3 and g(x) = 3x + 5. Find

   i) $f^{-1}{}_o g(x)$ ii) $f_o g^{-1}(x)$ iii) $(f_o\, g)^{-1}(x)$
   iv) $f^{-1}{}_o g^{-1}(x)$ v) $g_o f(x)$ vi) $(g^{-1}{}_o\, f)(x)$

8. Suppose the total cost in dollars of manufacturing q units of a certain commodity is given by the function $C(q) = q^3 – 30q^2 + 400q + 500$. Compute the cost of manufacturing 20 units.

Lecture – 24 (50 minutes)

Quadratic Equations and Functions

a) Factorization and use of quadratic formula to solve a quadratic equation.
b) Class works

A. Factorization and use of quadratic formula to solve a quadratic equation.

Various properties of quadratic equation may be derived directly from those of the general equation of degree *n.* But, in many occasions, we do not need the general theory of equation of degree *n.* It is generally felt that a systematic study of the theory of quadratic equation throws sufficient lights on the general theory. We shall therefore consider the quadratic equation in a greater detail.

A quadratic equation is in the form $ax^2 + bx + c = 0$, here $a \neq 0$, if a = 0, the equation becomes linear. A quadratic equation has two values of x which are known as the roots of the equation. The roots can be obtained by various methods. We discuss on the factorization and use of formula methods.

Factorization method.
In this method, we factorize the given quadratic equation to obtain two factors of the equation whose product equals to zero. Taking each factor equal to zero turn by turn, we calculate the possible values of x. i. e. roots of the equation. Let us see an example.

Example:
Solve $2x^2 - 5x + 3 = 0$.
Solution:
We know how to factorize a quadratic function. we first find the product of constant term and coefficient of x^2. Then finding the factors of the product we find two numbers whose sum or different is in the middle term (as coefficient of x).
here the product of 2 and 3 is 6. whose factors are 1 and 6 or 2 and 3. to get 5 by the sum (because the constant term is positive), we use 2 and 3 and find factors.

$2x^2 - 5x + 3 = 0$
or $2x^2 - 3x - 2x + 3 = 0$
or $x(2x - 3) - 1(2x - 3) = 0$
or $(2x - 3)(x - 1) = 0$
either $2x - 3 = 0$ .....(1) or $x - 1 = 0$........ (2)

From (1) $x = 3/2$ and
From (2) $x = 1$.

The same problem can be solved using a formula. The required formula is derived in the following theorem.

Theorem 1.

The two roots of a quadratic equation,

$ax^2 + bx + c = 0$ are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

proof:

We have the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$.

$\Rightarrow x^2 + (b/a)x + (c/a) = 0$ [since $a \neq 0$, we can divide all by a]

$\Rightarrow x^2 + 2.x.(b/2a) + (b/2a)^2 = (b/2a)^2 - (c/a)$

$\Rightarrow (x + b/2a)^2 = (b^2 - 4ac)/4a^2$

$$\Rightarrow x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$$

$$\therefore x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} \text{ Proved.}$$

Example:
Solve $2x^2 – 5x + 3 = 0$ using the formula.
Solution:
First we compare the given equation with $ax^2 + bx + c = 0$.
so, a = 2, b = -5 and c = 3
Now by using the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4.2.3}}{2.2}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{4}$$

$$= \frac{5 \pm 1}{4}$$

taking positive sign, we get $x = \frac{5+1}{4}$ = 6/4 = 3/2

taking negative sign we get $x = \frac{5-1}{4}$ = 4/4 = 1.

B. Class Work

Solve the following quadratic equations by both the methods: use of formula and factorization and check whether the answers are same or not.

i) $x^2 + 4x + 3 = 0$ ii) $3x^2 - 8x + 4 = 0$ iii) $2x^2 + 5x - 3 = 0$ iv) $x^2 - 4x + 4 = 0$