# Normal Distributions <br> Lecture 6 

## Intro to normal distributions



## Intro to normal distributions



Normal distribution differing in mean and standard deviation

# Intro to normal distributions 

$$
\frac{1}{\sqrt{2 \pi \sigma^{2}}} e^{\frac{-(x-\mu)^{2}}{2 \sigma^{2}}}
$$

## 7 features of normal distributions

- Normal distributions are symmetric around their mean.
- The mean, median, and mode of a normal distribution are equal.
- The area under the normal curve is equal to 1.0.
- Normal distributions are denser in the center and less dense in the tails.
- Normal distributions are defined by two parameters, the mean $(\mu)$ and the standard deviation ( $\sigma$ ).
- $68 \%$ of the area of a normal distribution is within one standard deviation of the mean.
- Approximately 95\% of the area of a normal distribution is within two standard deviations of the mean.


## History of the normal distribution

$$
P(x)=\frac{N!}{x!(N-x)!} \pi^{x}(1-\pi)^{N-x}
$$

## History of the normal distribution



Examples of binomial distributions. The heights of the blue bars represent the probabilities.

## History of the normal distribution



The normal approximation to the binomial distribution for 12 coin
flips. The smooth curve is the normal distribution. Note how well it
approximates the binomial probabilities represented by the heights of the blue lines

## History of the normal distribution

## areas under normal distributions



Normal distribution with a mean of 50 and standard deviation of
$10.68 \%$ of the area is within one standard deviation (10) of the mean (50).

## areas under normal distributions



Normal distribution with a mean of 100 and standard deviation of
$20.68 \%$ of the area is within one standard deviation (20) of the mean (100).

## areas under normal distributions



A normal distribution with a mean of 75 and a standard deviation
of $10.95 \%$ of the area is within 1.96 standard deviations of the mean

## Standard normal distribution

| $Z$ | Area below |
| ---: | ---: |
| -2.5 | 0.0062 |
| -2.49 | 0.0064 |
| -2.48 | 0.0066 |
| -2.47 | 0.0068 |
| -2.46 | 0.0069 |
| -2.45 | 0.0071 |
| -2.44 | 0.0073 |
| -2.43 | 0.0075 |
| -2.42 | 0.0078 |
| -2.41 | 0.008 |
| -2.4 | 0.0082 |
| -2.39 | 0.0084 |
| -2.38 | 0.0087 |
| -2.37 | 0.0089 |
| -2.36 | 0.0091 |
| -2.35 | 0.0094 |
| -2.34 | 0.0096 |
| -2.33 | 0.0099 |
| -2.32 | 0.0102 |

## Standard normal distribution

Normal Distribution


Shaded area: 0.006210
An example from the applet

## Standard normal distribution

$$
Z=(X-\mu) / \sigma
$$

## Standard normal distribution

| $Z$ | Area below |
| ---: | ---: |
| -2.5 | 0.0062 |
| -2.49 | 0.0064 |
| -2.48 | 0.0066 |
| -2.47 | 0.0068 |
| -2.46 | 0.0069 |
| -2.45 | 0.0071 |
| -2.44 | 0.0073 |
| -2.43 | 0.0075 |
| -2.42 | 0.0078 |
| -2.41 | 0.008 |
| -2.4 | 0.0082 |
| -2.39 | 0.0084 |
| -2.38 | 0.0087 |
| -2.37 | 0.0089 |
| -2.36 | 0.0091 |
| -2.35 | 0.0094 |
| -2.34 | 0.0096 |
| -2.33 | 0.0099 |
| -2.32 | 0.0102 |
|  |  |



Shaded area: 0.008198
Area below 26 in a normal distribution with a mean of 50 and a standard deviation of 10

Normal approximation to the Binomial


Approximating the probability of 8 heads with the normal distribution

## Normal approximation to the Binomial



Shaded area: 0.986574

Area below 8.5

## Normal approximation to the Binomial



## Shaded area: 0.943081

Area below 7.5

## Calculating without calculator

- Find a $Z$ score for 8.5 using the formula $Z=(8.5-5) /$ $1.5811=2.21$.
- Find the area below a $Z$ of $2.21=0.987$.
- Find a Z score for 7.5 using the formula $Z=(7.5-5) /$ $1.5811=1.58$.
- Find the area below a $Z$ of $1.58=0.943$.
- Subtract the value in step 4 from the value in step 2 to get 0.044.
- Thanks for attention

