Duality

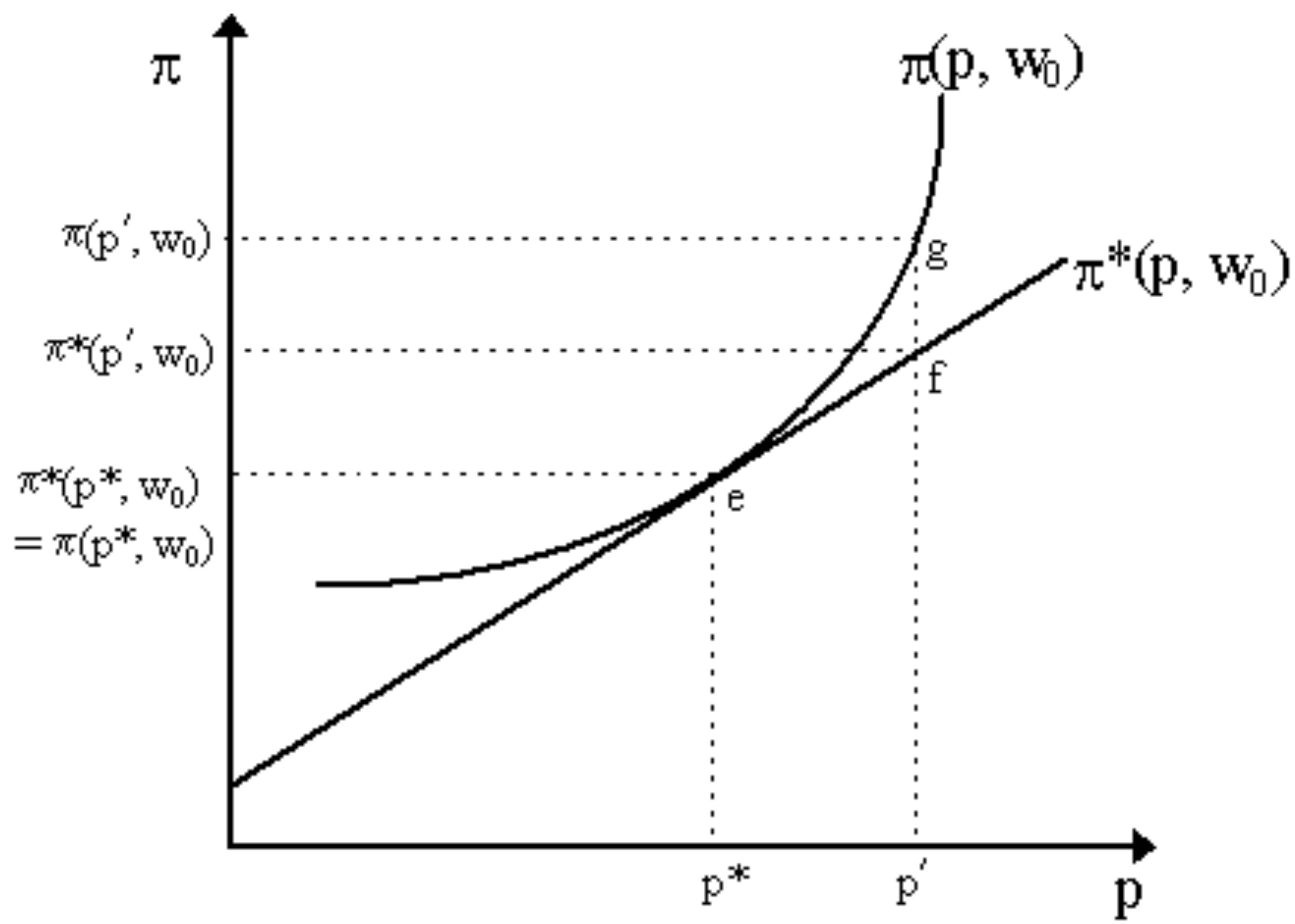
Background

Duality

cost, profit and production

$$\sup_{(p,w)\in\mathbf{R^N}} p\cdot x + w\cdot y - \pi(p,w) = \begin{cases} 0 & \text{if } (x,y)\in F, \\ +\infty & \text{otherwise.} \end{cases}$$

Hotelling's lemma



Profit function

$$\frac{\partial y_k}{\partial p_l} = \frac{\partial^2 \pi}{\partial p_k \partial p_l} = \frac{\partial^2 \pi}{\partial p_l \partial p_k} = \frac{\partial y_l}{\partial p_k}$$

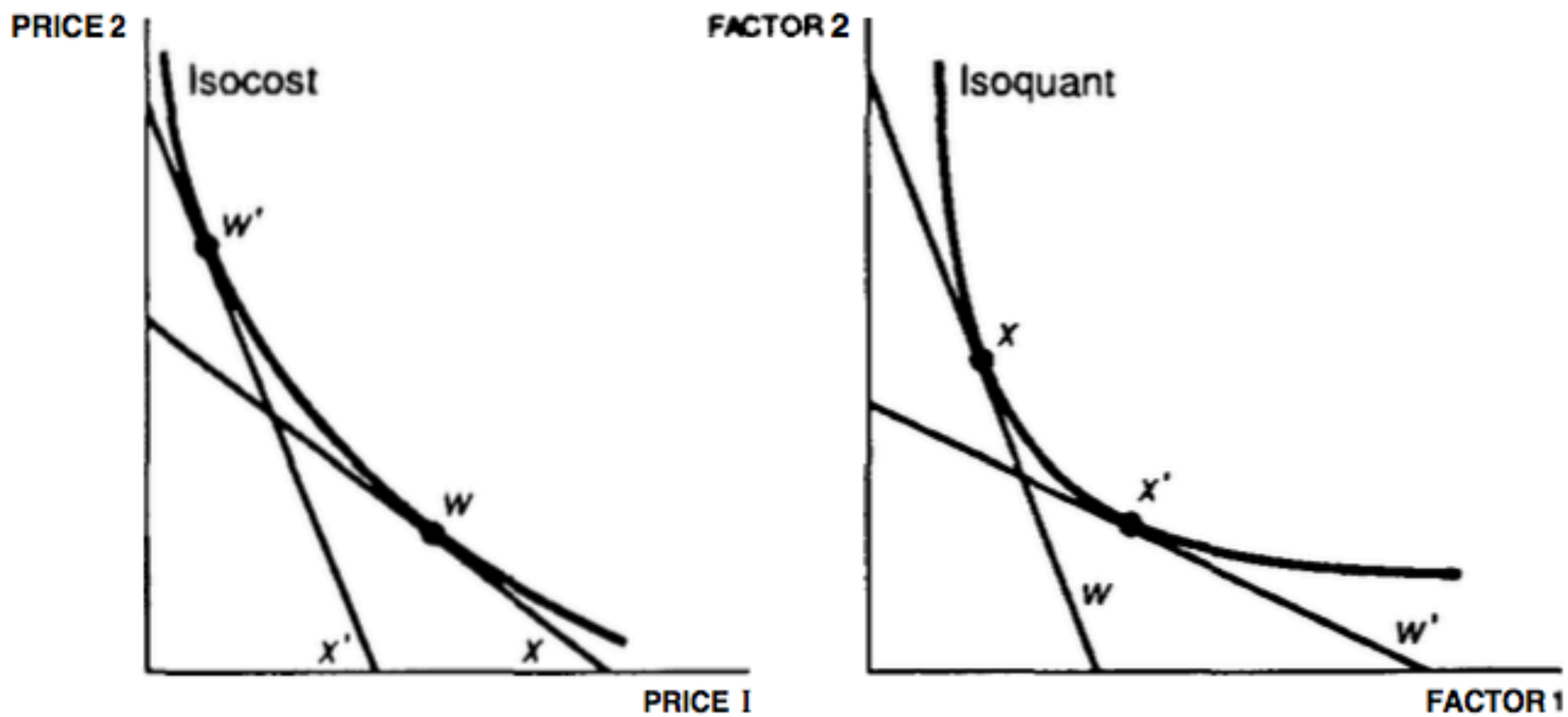
$$p_i \cdot (y_j - y_i) + p_j \cdot (y_k - y_j) + \cdots + p_l \cdot (y_i - y_l) \leq 0.$$

Shephard's lemma

$$h_i(\mathbf{p}, u) = \frac{\partial e(\mathbf{p}, u)}{\partial p_i}$$

$$x_i(\mathbf{w}, y) = \frac{\partial c(\mathbf{w}, y)}{\partial w_i}$$

Geometry of duality



$$\frac{dw_2(w_1^*)}{dw_1} = -\frac{\frac{\partial c(\mathbf{w}^*, y)}{\partial w_1}}{\frac{\partial c(\mathbf{w}^*, y)}{\partial w_2}} = -\frac{x_1(\mathbf{w}^*, y)}{x_2(\mathbf{w}^*, y)}.$$

Geometry of duality

$$f(\mathbf{x}) \equiv y$$

$$\frac{dx_2(x_1^*)}{dx_1} = -\frac{\frac{\partial f(\mathbf{x}^*)}{\partial x_1}}{\frac{\partial f(\mathbf{x}^*)}{\partial x_2}}$$

$$\frac{w_1^*}{w_2^*} = \frac{\frac{\partial f(\mathbf{x}^*)}{\partial x_1}}{\frac{\partial f(\mathbf{x}^*)}{\partial x_2}}$$