

# Cost minimization

# Cost minimization problem



# Long run vs Short run

Short run:  $q = f(L)$

Long run:  $q = f(K, L)$

# Choice of production process



Short run:  $q = f(L)$   
Long run:  $q = f(K, L)$

# Deciding the cheapest production



# Cost minimization rule

*cost minimized where*  $\frac{MP_L}{w} = \frac{MP_K}{r}$

# Cost minimization

$$\frac{MP_L}{w} > \frac{MP_K}{r}$$

*L ↑ and K ↓ to reduce cost*

$$\frac{MP_L}{w} \downarrow \text{ and } \frac{MP_K}{r} \uparrow$$

*Repeat until balance achieved!*

# Lagrange multipliers

$$\mathcal{L}(\lambda, \mathbf{x}) = \mathbf{w}\mathbf{x} - \lambda(f(\mathbf{x}) - y)$$

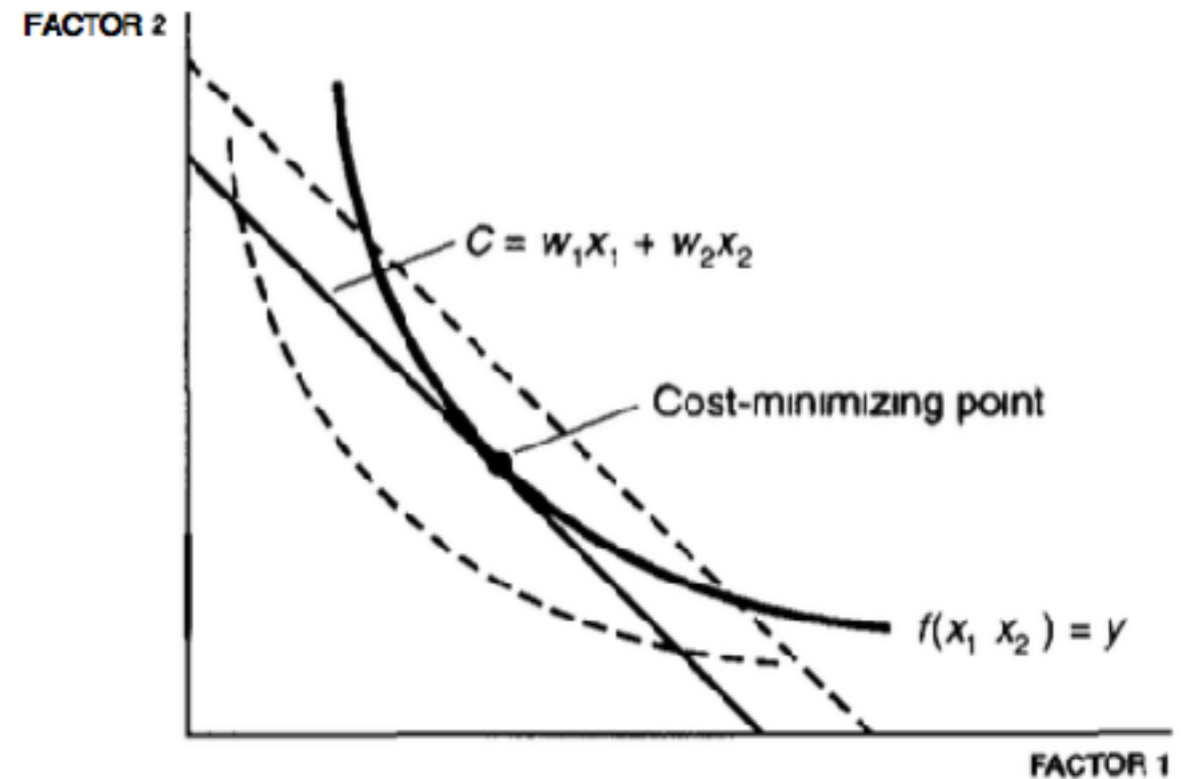
$$w_i - \lambda \frac{\partial f(\mathbf{x}^*)}{\partial x_i} = 0 \quad \text{for } i = 1, \dots, \mathbf{n}$$
$$f(\mathbf{x}^*) = y$$

$$\mathbf{w} = \mathbf{A} \mathbf{D}f(\mathbf{x}^*)$$

$$\frac{w_i}{w_j} = \frac{\frac{\partial f(\mathbf{x}^*)}{\partial x_i}}{\frac{\partial f(\mathbf{x}^*)}{\partial x_j}} \quad i, j = 1, \dots, n$$

# Cost minimization

$$\frac{w_i}{w_j} = \frac{\frac{\partial f(\mathbf{x}^*)}{\partial x_i}}{\frac{\partial f(\mathbf{x}^*)}{\partial x_j}} \quad i, j = 1, \dots, n$$



$$f(x_1 + h_1, x_2 + h_2) \approx f(x_1, x_2) + (f_1 \quad f_2) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \\ + \frac{\mathbf{1}}{2} (h_1 \quad h_2) \begin{pmatrix} f_{11} & \hat{f}_{12} \\ f_{21} & f_{22} \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$